The Optimal Time for Claiming Social Security Benefits: A Methodological Note

by

Joseph Friedman
Department of Economics
Temple University

Department of Economics
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1301 Cecil B. Moore Avenue, Philadelphia, PA 19122

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Joseph Friedman
Department of Economics
Temple University
Philadelphia, PA 19122

jfriedman@temple.edu

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Abstract

The initiation of benefits versus postponement of benefits decision and the optimal age for initiating Social Security benefits are subjects of a number of recent papers. It is generally agreed that an initiation versus postponement of benefits decision may have significant consequences, but there is less agreement on how to model the problem or measure its financial implications.

By law, benefits are paid only to live beneficiaries. Thus, the anticipated future benefits should be weighted by the recipient's survival probabilities – the probabilities that the recipient is alive when the benefits will actually be received. Many published papers assume that benefits will be received "on average" throughout the recipient's expected remaining lifetime and estimate the present value of Social Security benefits by discounting the cash flow through life expectancy.

This paper will show the preferred approach is to estimate the Actuarial Present Value (APV) which weighs each future payment by the probability that it will be received. Based on survival probabilities and life expectancy tables that

are compiled by the CDC the paper demonstrates that the present value through life expectancy approach overstates the APV by approximately 10%   Therefore, timing decisions that are not based on the APV are probably suboptimal.

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Social Security Benefits (SSB) may be initiated at any age between 62 and 70. Retirees who choose to initiate SSB at a younger age, all other things equal, will receive smaller benefits than those who postpone initiation of SSB to a later age. On the other hand, early initiators receive their reduced SSB for a longer period. The optimal timing for initiating SSB has been the subject of many recent papers. It is generally agreed that the timing of initiation versus postponement of benefits decision may have significant consequences, but there is less agreement on how to model the problem or measure its financial implications. All recent papers use present value (PV) calculation to compare and contrast alternative future cash flows but they differ in the way they account for the fact that SSB are paid only to live recipients who have uncertain lifetime[1].

SSB are ***Whole-Life-Annuities*** – the benefits are paid monthly for as long as the beneficiary survives[2]. Since SSB are paid only to live recipients, each anticipated future payment should be weighted by the probability that it will be actually received. That is, future payments should be weighed the probability that

[1] See Docking, Fortin and Michelson (2013) for a recent literature review.
[2] Actuaries call an annuity "whole-life" when payments are made for as long as the beneficiary survives. That is, survival is a condition for receiving payment. Most readers of this *Review* are familiar with "annuity-certain" which has a fixed (certain) time horizon.

the recipient or his/hers spouse will still be alive at the beginning of the period in which a payment is due. Thus, the expected present value of any future payment is determined not only by its discount rate assumption and cash flow definitions, but also by the time horizon over which payments is assumed to be received.

The correct method to calculate the present value of SSB is the ***Actuarial Present Value*** (APV), also known as the ***Expected Present Value***. The seminal paper that uses APV in economics (for the general case of demand for annuities) is Yaari (1965), but the actuarial literature is much older[3]. The APV is computed by multiplying each present value of a future SSB payment by the probability that it will be received and all the products are then added up. Examples of papers using the APV are Munnell and Soto (2007), Coile, Diamond, Gruber and Justen (2002) and Friedman and Phillips (2008 and 2010).

Many papers assume that a recipient will receive SSB "on average" throughout his or hers Expected Remaining Lifetime (ERL) and estimated the present value of an Annuity Certain with a time horizon that equals the ERL. As Jordan (1967) states this is a "persistent misconception" and proves mathematically that the APV of a life-annuity at age x is in fact ***smaller*** than "the present value of a life annuity certain for a term equals to the life expectancy at age *x* ", see Jordan 1967, p.174. Examples of papers using this approach are

[3] See for example de Witt (1670-1672) and Allen (1907)

Fraser, Jennings, and King (2000), McCormack and Perdue (2006), Spitzer (2006), Docking et al (2013) and many others.

This paper explains why the APV method is the preferred method when compared to Annuity Certain with ERL as the time horizon (ACLE). Using survival probabilities and life expectancy tables that are compiled by the Center for Diseases Control (CDC) this paper demonstrates that the ACLE approach overstates the APV by approximately 10%. It would follow, therefore, that decisions based on ACLE are not optimal.

The ACLE approach is even more problematic when the present value of SSB of ***married couples*** is evaluated. Depending on their earning histories, married couples can chose to receive SSB as two single individuals, as an individual and his/hers spouse, or as a surviving spouse (if the other spouse is deceased). For the case of married couples this paper will show the expected value of each period's SSB depends on the probability that the spouse A and spouse B are jointly alive, the probability that spouse A is alive and the spouse B is dead and the probability that spouse B is alive and spouse A is dead. Thus, the APV cannot simply be approximated by the sum of spouse A and spouse B ACLE as was done, for example, by McCormack and Perdue op. cit. and Docking, at al, op. cit.

It should be noted that the SSB of married couples is a joint-life annuity, with benefits received until the second spouse dies. Thus, the life expectancy that matters is the couple's joint life expectancy, not the spouses' life expectancies as individuals: For example, as is shown in the Appendix tables (which are based on data from the CDC), the ERL of a 66 years old male is 16.9 and the ERL of a 66 years old female is 19.5, the joint life expectancy of a married couple (where both husband and wife are 66) is 25.3 years -- one of them is likely to survive that long[4]. Moreover, as is shown in the Appendix and consistent with the formal definition of ERL, the probability that a person will outlive its life expectancy is 50% (that is, 50% of people die before their ERL and 50% die after their ERL). For a married couple the probability that at least one member of a couple will outlive his/hers life expectancy 75%.[5]

This paper proceeds as follows: Section 2 discusses the difference between the APV and ACLE methods for a single retiree and for married couples. Section 3

[4] Data for joint life expectancy is from IRS publication 590, Appendix C, Table II.

If an individual has 50% chance to die before life expectancy and mortality of the husband is independent of the mortality of the wife (that is, there is no "broken heart syndrome.") Then the probability that both die before their respective life expectancies is 0.5x.0.5 =0.25. The complement event is that at least one person in a couple lives longer than his/hers life expectancy. The probability of this event is 1-0.25 = 0.75. An alternative explanation is: The sample space has 4 points with equal probabilities: (S,S) (S,F) (F,S) (F,F), where S is success (i.e. surviving over the expected life) and F is failure; first coordinate for husband and second for wife. The "event" is at least one S, this occurs in 3 out of 4 points, and hence the probability is ¾.

presents numerical examples. An application of the APV models to the SSB claiming of married couples is presented in Section 4. Summery and conclusions are presented in Section 5.

2. Estimating Expected Present Value

2.1 A single retiree:

We need to determine the value of an income stream that is contingent on the recipient being alive. Consider the following example. Sally is trying to determine when she should start collecting Social Security. Her latest Social Security statement shows that if she initiates SSB at her Normal Retirement AGE, 66, her monthly benefit will be $1,000. If she initiates at age 62, her monthly benefit will be $750, and if she postpones initiation to age 70 her monthly benefit will be $1,320. She has enough income from other sources that she can wait the extra eight years if it would be more beneficial to her. What should Sally do? On the one hand, she receives a much higher benefit if she waits. On the other hand, she might die before or soon after she reaches age 70. She can use a financial calculator or an Excel spreadsheet and compute the present value of the three alternative streams of payments, but how should she take her mortality into account? This is explained below.

Because the available Life Tables provide survival probabilities only for integer ages, the author will assume that the benefits are paid once a year at the beginning of each year[6]. Let *a(x,r)* denote the actuarial present value of $1 to be received each year for as long as the recipient is alive, where x = the recipient's current age, and r = real interest rate of r per annum. Then *a(x,r)* is calculated as

$$APV = a(x,r) = \$1 \cdot \sum_{t=x}^{\Omega} p(x,t) \cdot v^t \qquad (1)$$

Where $p(x,t)$ is the probability that an individual aged x will be alive at age t, $v^t = \frac{1}{(1+r)^t}$ is the discount factor and Ω = the oldest age in the life table (100).

$p(x,t)$ is defined by

$$p(x,t\ ) = \prod_{t=0}^{t-1} (1 - q_{x+i}) \qquad (2)$$

Where q_x is the probability of dying between age x and x+1, (q_x is obtained from the life tables, such as those in Appendix A.)

Equation (1) is often *approximated* by Equation (3), the ACLE, by arguing that income will be received **on average** through the ERL.

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[6] The annual $p(x,t)$, can be converted to monthly probabilities by assuming that people are dying at a constant monthly rate between year t and year t+1. Increasing the granularity of the life table will make the calculations more cumbersome but will not affect the main results.

$$ACLE = v(x,r) = \sum_{t=x}^{E(Tx)} v^t \qquad (3)$$

In Equation (3) E(Tx) is the ERL and T_x is the remaining lifetime of an individual aged x and $v^t = \frac{1}{(1+r)^t}$.

According to Bowers, et al (1986, pp 149-150), Jordan (1967, p.174) and Milevsky (2006, p. 116), the approximation of the APV by using Equation (3) will overstate the APV, that is, $v(x,r) > a(x,r)$. This fact is a corollary of ***Jensen's Inequality,*** a well-known mathematical theorem.

The relationship between Equation (1) and Equation (3) can be seen if a second term (whose value is zero) is added to the right-hand side of Equation (3). The second sum in Equation (4) is redundant but is included for ease of exposition:

$$v(x,r) = \sum_{t=x}^{E(Tx)} 1 * v^t + \sum_{t=E(Tx)}^{\Omega} 0 * v^t \qquad (4)$$

Comparing Equation (1) with Equation (4) one can see that the ACLE approximation requires two assumptions that are rarely stated:

(i) All the payments until age $E(T_x)$ will be received with certainty, that is

$p(x,t)=1$ for t smaller or equal to $E(T_x)$.

(ii) No payments will be received past $E(T_x)$; that is **p**$(x,t)=0$ *for t greater than* $E(T_x)$.

The magnitude of the overstatement (bias) is shown in Table 3.1, below.

2.2 Married Retirees

. By SSA rules, married people can choose to receive SSB based on their own record or 50% of their spouse SSB, whichever amount is larger. When one spouse passes the surviving spouse can choose to receive survival benefits. Continuing our example from the previous section: Assume that Sally is married to Peter who never worked for a living. Computing the APV of the couple's SSB is somewhat more complicated since Peter is eligible to receive spousal benefits of 50% of Sally's SSB as long as they are both alive and if Peter survives Sally he will be eligible to receive 100% of her SSB. The APV of a joint and survivor annuity with a 'last survivor provision' is given in Equation 5, below. Equation 5 was adopted from Brown and Porteba (1999).

Let B denote the fixed periodic SSB that Sally would have received had she been single. The SSB paid to the couple as long as both spouses are alive is 1.5B. After one spouse passes the surviving spouse will continue to receive 1B per period. Let $p_m(x,t)$

denote the probability that the husband aged x will be alive at time t and $p_f(x,t)$ denote the probability that the wife aged x will be alive at time t . The APV for the joint and survivor annuity contract is given by Equation (5):

$$APV = \sum_{t=x}^{\Omega} \{0.5B * P_m(x,t) * P_f(x,t) + B\left[P_m(x,t) + P_f(x,t) - P_m(x,t) * P_f(x,t)\right]\} * v^t \qquad (5)$$

In Equation (5) the term $P_m(x,t) * P_f(x,t)$ is the probability that both husband and wife are jointly alive, at time t. The term in the square brackets is the probability that at least one member of the couple is still alive at time t.

Since using Equation (5) is somewhat tedious some researchers chose to evaluate the Social Security wealth of married couples by adding the ACLE of the husband to the ACLE of the wife. In addition to the bias noted earlier, this approach is problematic for another reason: While life expectancy is the average future life of an individual approximately 50% of men and 50% of women will outlive their life expectancy. However, the probability that the husband and wife will *both* die before their reach their respective life expectancies is only 25% and the probability that *at least* one spouse will outlive his or her ERL is 75%. Thus, estimating the value of SSB for couples using the ACLE will be incorrect for most couples. A numerical example for married couples is shown in Table 3.2, below.

3. Numerical Examples

3.1 Single Individuals

Table 3.1 contrasts the results of APV and the ACLE calculations for a 66 years old retiree whose SSB are $1,000 per month. Because the US Life Tables are tabulated for integer years the calculations, it is assumed in the calculations that SSB are received as a single payment of $12,000 per year. The APV and ACLE values shown in the table were computed for an assumed 2% real interest rate, and life expectancy (ERL) of 66 years old males is 16.9 and of 66 years old females is 19.5. The results are consistent with the Jensen Inequality theorem which predicts that ACLE overstates APV. Therefore, , consistent with economic theory, if one accepts that APV is the correct way to evaluate life-annuities then one must conclude that models based on ACLE incorrect.

[PLEASE INSERT TABLE 3.1 HERE]

3.2 Married Couples

3.2.1. Heterosexual Marriage

We now return to the example of Sally and Peter who were introduced in Section 2.2. Both Sally and Peter, the reader will recall, are 66 years old, Sally is the retiree and Peter is her never-employed spouse. Based on these assumptions, as long as they are both alive they will together receive 150% of Sally's SSB, but after one of them dies the surviving spouse will receive 100% of Sally's SSB.

For the ACLE calculations one needs to assume that the two spouses will live ***exactly*** to their respective expected remaining lives, but as stated earlier the probability of this happening is only 25%. For Sally and Peter their Social Security ACLE are obtained from Table 3.1: As long as one of them is alive the couple will receive Sally's SSB of $196,798 ***plus*** as long as Peter is alive the couple will receive his spousal benefits which are half of $179,217, or $89,609. Using the ACLE method the couple's Social Security wealth is, therefore, $268,826. It is important to note that the preceding calculation assumes that Peter will die before Sally. If Sally dies before Peter he will be eligible to collect benefits as a surviving spouse but the ACLE method is not suitable for such cases.

[PLEASE INSERT TABLE 3.2 HERE]

The APV calculations are more involved since they require the application of Equation 4 of section 2. The APV calculations do not require any assumption about the order in which each spouse will die and for how long the surviving spouse will live as a widow or widower. Additionally, the APV can accommodate the surviving spouse benefits. The corresponding Social Security wealth result for Sally and Peter is $295,699 (see Table 3.2).

4. Application: Should 66 Years Old Retirees Delay Claiming?

Tough Full Retirement Age (FRA) is presently 66, a retiree or a retired couple, who just turned 66 years of age may wish to consider postponing the initiation of SSB because for each month of postponement until age 70 their future benefits increase at a rate of 2/3% a month for each monthly postponement up to a maximum of 32% if benefits are not initiated until the age of 70. A complete analysis of the optimal initiation age is beyond the scope of this article. Instead, in this section, we demonstrate the use of the APV method to calculate the rate of return on a one-year postponement.

As in Friedman and Phillips (2008), the internal rate of return (IRR) on a one-year postponement is found using the "Solver" tool in Excel which searches for an IRR at which the value of the immediate annuity (the APV of starting benefits at 66) equals the value of the deferred annuity (the APV of starting and receiving 8% higher benefits at age 67). When benefits are postponed by one year, our couples forfeit the SSB of $18,000 that they would have received if they claimed at 66, but upon reaching age 67 they would be eligible for an 8% ($1,440) higher SSB for life (That is, until the second spouse passes). The IRR that was found through the iterative search is 1.65%, see Table 4.1. This IRR should be interpreted as a hurdle rate. That is, couples that can invest the age 66 SSB and earn a real rate of return higher than the IRR shown in table 4.1 should initiate SSB early. Couples that cannot earn the hurdle rate on their own investments may consider deferment

A word of caution is in order. Buying an annuity is not the same as buying a bond. An annuity provides lifetime income, an important consideration for healthy couples who can expect a long life. On the other hand, couples who are not healthy or couples with strong bequest motive may choose to ignore the hurdle rate. The reader is referred to Friedman and Phillips (2008) for a farther elaboration of this point.

5. Summary

Almost all American workers are entitled to receive Social Security benefits when they retire, and this entitlement represents an important source of retirement income for most retirees. The Social Security entitlement is often called Social Security Wealth by economists, see for example Martin Feldman (1976) who argued Social Security wealth must be added to fungible wealth when figuring out people's Total Wealth. This paper shows that the two methods of evaluating Social Security Wealth, i.e. the present value of Social Security retirement benefits lead to different results. The paper argues that calculations by *discounting the cash flow through life expectancy* overstate the theoretically correct calculations of *expected present value*. In addition to being the theoretically correct method, the APV is flexible enough to enable estimation of married couples whose Social Security Entitlement is in fact a joint and survivor annuity.

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TABLES

Table 3.1: Present Value of Social Security Benefits evaluated at Age 66

(Assuming 2% real discount rate)

	ERL[1]	APV a(x,r)	ACLE v(x,r)	Difference
Male	16.9	$172,155	$179,217	$7,062
Female	19.5	$193,023	$196,798	$3,775

[1] the ERL are from Arias, National Vital Statistics Reports, Vol.62, No.7, 2014

Table 3.2: Present Value of Social Security Benefits of Married Couples evaluated at Age 66

(Assuming 2% real discount rate)

APV a (x,r)	ACLE v(x,r)	Difference
$295,699	$268,826	$26,873

Table 4.1: Internal Rate of Retune on a one year Postponement of SSB from initiation at 66 to initiation at 67

Rate of Return	1.65%

Assumption: Married couple, decision age: 66