Estimating a Falsified Model: Some Impossibility Theorems

by

Andrew J. Buck
Department of Economics (emeritus)
Temple University

George M. Lady
Department of Economics
Temple University
george.lady@temple.edu

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Abstract

A recent literature, e.g., Lady and Buck (2011), has shown that a qualitative analysis of a model's structural and estimated reduced form arrays can provide a robust procedure for assessing if a model's hypothesized structure has been falsified. This paper shows that the even weaker statement of the model's structure provided by zero restrictions on the structural arrays can be falsified, independent of the proposed nonzero entries. When this takes place, multi-stage least squares, or any procedure for estimating the structural arrays with the zero restrictions imposed, will present estimates that could not possibly have generated the data upon which the estimated reduced form is based. The examples given in the paper are based upon a Monte Carlo sampling procedure that is briefly described in the appendix.

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Department of Economics

Temple University

Philadelphia, PA

Estimating a Falsified Model: Some Impossibility Theorems

I. Introduction. As early as Samuelson (1947), if not before, it was pointed out that economic theory often only specified the directions of influence among the endogenous and exogenous variables of an economic model, as represented by a proposed sign pattern for a model's structural arrays. Given this, the structure hypothesized by the theory could only be falsified if the proposed structure placed limits on the sign pattern of the corresponding reduced form. Both Samuelson (op. cit.) and the subsequent literature, e.g., Hale, *et al* (1999), proposed that this required that certain entries of the reduced form must take on particular signs, no matter what; and, if such signs were not estimated, then the structure was falsified. The analytic conditions under which the sign pattern of the structure would require specific signs for entries of the reduced form were extremely restrictive, virtually never satisfied, and such an analysis, called a qualitative analysis, is virtually never conducted. Recently, Lady and Buck (2011) and Buck and Lady (2012) showed that even if particular signs in the reduced form were not required by an hypothesized structure, nevertheless limits were always implied and any model so specified could be potentially falsified by the sign pattern of the estimated reduced form. Since current econometric practice does not call for the conduct of a qualitative analysis, a model's falsification in these terms would not be detected. Still, for a qualitatively falsified model, when the structural arrays are estimated in turn, e.g., via multi-stage least squares, the sign pattern of the estimated structural arrays would presumably be different from the hypothesized structural arrays, although it is not clear that this outcome must always take place. An immediate issue is whether or not the sign pattern of the estimated structure is consistent with that of the estimated reduced form. This paper shows that the zero restrictions on the hypothesized structure can be falsified independent of the characteristics of the nonzero entries. When this happens, the estimated structure with the zero restrictions imposed could not possibly have generated the data upon which it is based. Without conducting a qualitative analysis, current econometric practice can estimate structural arrays which are impossible, given the outcome of the estimated reduced form. This paper presents the basis for this circumstance and an algorithmic procedure for detecting such circumstances when they occur.

II. Background. It is assumed that a model of some aspect of the economy is expressed by a system of simultaneous equations:

$$f^{i}(Y, Z) = 0, i = 1, 2, \ldots, n, \qquad (1)$$

where Y is an n-vector of endogenous variables and Z is an m-vector of exogenous variables. The features of this system at issue in terms of establishing its validity are studied via the method of comparative statics. This method studies the changes in the solution values of the endogenous variables as brought about by changes in the values of the exogenous variables, all expressed by a linear system of differentials:

$$\sum_{j=1}^{n} \frac{\partial f^{i}}{\partial y_{j}} dy_{j} + \sum_{k=1}^{m} \frac{\partial f^{i}}{\partial z_{k}} dz_{k} = 0, i = 1, 2, \ldots, n. \quad (2)$$

When these systems are brought to the data it is typical to assume that (1) is at least locally linear, and (2) is re-expressed as:

$$\beta Y = \gamma Z, + \delta U, \qquad (3)$$

where β, γ and δ are appropriately dimensioned matrices, with δU representing errors embodied in the data. The system (3) is usually termed the *structural form* and the qualitative specification of the model is provided by the sign patterns of β and γ. The system is then brought to the data by its re-expression as the *reduced form*,

$$Y = \pi Z + \psi U, \qquad (4)$$

where $\pi = \beta^{-1}\gamma$. For all of this to be "scientific," it must be possible to show that the hypothesized specification of (1) and (2), as re-expressed as (3) somehow limits the outcome when estimating (4). As noted above, if only the sign patterns of the structural arrays are hypothesized, it was believed that such an hypothesis could only be falsified if at least some entries of the reduced form had to have particular signs, independent of the magnitudes of the entries of the structural arrays. The recent literature cited above showed that this was not so. The point here is to reiterate this point; and further, to show that under some circumstances, a falsified model can nevertheless be worked with to estimate the entries of the structural arrays, and that the estimated entries could not have possibly generated the data upon which they are based.

Monte Carlo Sampling: The analytical approach we take is based upon the Monte Carlo sampling procedures cited in this recent literature. This approach is spelled out in, e.g., Lady and Buck (op. cit.) and briefly reiterated here in the Appendix. The basic idea is to take sign patterns proposed for the structural arrays and, subject to distributional rules (uniform in the analysis reported on here) assign magnitudes to the entries and construct the reduced form. The sampling can be done millions of times. For a particular reduced form sign pattern, the procedure notes the instances when this sign pattern shows up for a sample. If a particular reduced form sign pattern is not found for very large samples, the implication is that it is not possible, i.e., the proposed structural sign patterns impose limits that exclude it. For sufficiently small structural arrays, e.g., with reduced forms no larger than 5 x 5, a complete enumeration of all reduced form sign patterns found by the sampling can be tabulated. The results below are based upon this sampling approach.

III. A Signable Example. For the first example, it will be assumed that $\gamma = I$ and,

$$\text{sgn}\,\beta = \begin{bmatrix} - & - & 0 & - \\ + & - & - & 0 \\ 0 & + & - & - \\ + & 0 & + & - \end{bmatrix} \quad (5).$$

For felicitous presentation we will use "+" and "-" to denote the signs of entries, rather than the proper "1" and "-1." This sign pattern was presented in Lady and Maybee (1983) (because no column of β^{-1} could be entirely signed) . They provided a qualitative analysis that showed that twelve of the sixteen signs in the reduced form must take on particular signs for this sign pattern, i.e., for $\pi = \beta^{-1}$,

$$\text{sgn } \beta^{-1} = \begin{bmatrix} - & + & ? & + \\ - & - & + & ? \\ ? & - & - & + \\ - & ? & - & - \end{bmatrix} \quad (6).$$

In (6), the entries marked "?" were unsignable, and from the perspective of Lady and Maybee, they could take on any nonzero signs. Accordingly, they assumed, for the given sign pattern for β in (5) that there were sixteen possible outcomes for the estimated sgn π that would have been consistent with, i.e., would not falsify, the hypothesized sign pattern, given that γ = I. This circumstance, if it related to a small, applied model, would be a great (and unusual) victory for traditional qualitative analysis. Out of 65,536 possible sign patterns for the 4 x 4 reduced form (barring zeros), only sixteen were possible if the hypothesis (5) were true.

The point here is that even in this small, and signable example, problems remained. In particular, when a Monte Carlo sampling of the sign pattern (5) was conduced in Buck and Lady (2012), two of the sixteen supposed consistent sign patterns for β^{-1} where not found. These were,

$$\text{sgn } \beta^{-1} = \begin{bmatrix} - & + & -^* & + \\ - & - & + & +^* \\ +^* & - & - & + \\ - & -^* & - & - \end{bmatrix}; \textit{and},$$

$$\text{sgn } \beta^{-1} = \begin{bmatrix} - & + & +^* & + \\ - & - & + & -^* \\ -^* & - & - & + \\ - & +^* & - & - \end{bmatrix}.$$

The entries marked "*" are the supposedly unsignable entries of (6). And, they are unsignable; however; Buck and Lady (op. cit.) went on to show that, nevertheless, these two sign patterns could not possibly be taken on in (6), if the hypothesized (5) were true. The point there, and here, is that even though entries of the reduced form are not signable, patterns of signs may still be impossible, and if are the case for the estimated reduced form, then the hypothesized structural sign pattern has been falsified.

Still, in an applied context, if the estimated reduced form sign patterns were one of the two above, when an estimate of β was derived, e.g., via some form of multi-stage least squares, the estimated sign pattern would likely have differed from that hypothesized in (5), and the underlying qualitative analysis would not really have been necessary to falsify the hypothesis. Since the model is over-identified, an issue is whether or not the sign pattern of the estimated structure could possibly have generated the estimated reduced form. And the answer is: maybe not. When estimates of the structural arrays are derived, the hypothesized zero restrictions (and possibly other cardinal values, see Klein's model 1 below) are imposed. The point here is that the zero restrictions themselves can be falsified; and, when so, any proposed structure with these restrictions imposed could not possibly have generated the data upon which the estimates are based.

For the case here, Monte Carlo sampling was conducted for the case that $\gamma = I$ and,

$$\operatorname{sgn} \beta = \begin{bmatrix} ? & ? & 0 & ? \\ ? & ? & ? & 0 \\ 0 & ? & ? & ? \\ ? & 0 & ? & ? \end{bmatrix} \quad (5^*).$$

In constructing the samples, the entries marked "?" in (5*) were set positive or negative randomly, with equal probability. Tens of millions of samples were constructed and of the possible 65,536 sign patterns for the 4 x 4 reduced form, only 47,807 were found. Accordingly, (assuming the Monte Carlo results are valid) if any of the 17,729 sign patterns not found had resulted for the estimated reduced form, then <u>any</u> estimated structural arrays with the zero restrictions imposed would have been impossible, i.e., the estimated structure could not possibly have generated the data upon which the estimated reduced form is based: The estimated model would be an impossibility. For this case, an example is given in the last section.

IV. Klein's Model I. Klein's model I (Klein (1950)) is an over-identified model that often has appeared in the literature for a variety of methodological and pedagogical reasons, including its use for a qualitative analysis. For this last, Maybee and Weiner (1988) and later Lady (2000) studied the relationship between the structural arrays and the corresponding reduced form. It was also subjected to a qualitative analysis in Lady and Buck (op. cit.). Although given Klein's hypothesized sign patterns for the structural arrays, no entries in the corresponding reduced form were signable, some entries were "almost" signable. That is, when the expansions of the determinant and corresponding cofactors were studied, it worked out that most of the terms were of the same sign. A small number of additional assumptions about the relative size of a small number of entries, e.g., that the marginal propensity to consume was positive, but less than one, could be shown to result in the sum of the terms in the expansions at issue turning out to have a particular sign. Searching for signable entries in the reduced form based on additional, quantitative "side conditions" has a long history in conducting a qualitative analysis, e.g., Ritshard (1983) and Lady (1995), since entries are virtually never signable otherwise.

Suppressing the error vector, Klein's model is expressed as,

$$\beta Y = \gamma Z \text{, where}$$

$$\begin{bmatrix} -1 & 0 & a_1 & 0 & a_2 & 0 & 0 \\ 0 & -1 & 0 & 0 & b_1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & c_1 \\ 1 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} C \\ I \\ W_1 \\ Y \\ P \\ W \\ E \end{bmatrix} = \begin{bmatrix} -a_1 & -a_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & -b_2 & -b_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c_2 & -c_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} W_2 \\ P_{-1} \\ K_{-1} \\ E_{-1} \\ Year \\ TX \\ G \end{bmatrix}$$

In this structure the endogenous variables are private consumption (C), investment (I), the private wage bill (W_1), income (Y), profits or nonwage income (P), the sum of private and government wages (W), and private product (E); and the exogenous variables are the government wage bill (W_2), lagged profits (P_{-1}), end of last period capital stock (K_{-1}), lagged private product (E_{-1}), years since 1931 (Year), taxes (TX), and government consumption (G).

The sign patterns of the arrays proposed by Klein are as follows,

$$\operatorname{sgn}\beta = \begin{bmatrix} - & 0 & + & 0 & + & 0 & 0 \\ 0 & - & 0 & 0 & + & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & + \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix} \text{ and } \operatorname{sgn}\gamma = \begin{bmatrix} - & - & 0 & 0 & 0 & 0 & 0 \\ 0 & - & + & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & - & - & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & + & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}. \quad (7)$$

As an example, the (sign pattern) of the estimated reduced form of Klein's model is reported in Goldberger (1964) as

$$\textit{Estimated } \operatorname{sgn}\hat{\pi} = \begin{bmatrix} - & + & - & + & + & - & + \\ - & + & - & - & + & - & + \\ - & + & - & + & + & - & + \\ - & + & - & + & + & - & + \\ - & + & - & + & + & - & + \\ - & + & - & + & + & - & + \\ - & + & - & + & + & - & + \end{bmatrix}. \quad (8)$$

Although Goldberger did not know this, Lady and Buck (op. cit.) showed that the outcome (8) was not possible for the hypothesized structure (7), based upon a qualitative analysis. Nevertheless, the nine behavioral entries in (7) can be estimated, based upon the quantitative version of (8). Presumably, some of the signs in the estimated structure would have been different than the hypothesized signs in (7). Nevertheless, since the model is over-identified, even the signs in the estimated structure could be different, depending upon the estimation methodology utilized, and no alarm would be taken, e.g., Berndt (1996).

Our point is that the outcome of the estimated sign pattern (8) provides a decisive falsification of (7); namely, that regardless of the signs of the nine behavioral entries of (7), the sign pattern of the estimated reduced form given in (8) is impossible. In particular, for the structural arrays proposed as (7*),

$$\operatorname{sgn}\beta = \begin{bmatrix} - & 0 & ? & 0 & ? & 0 & 0 \\ 0 & - & 0 & 0 & ? & 0 & 0 \\ 0 & 0 & - & 0 & 0 & 0 & ? \\ + & + & 0 & - & 0 & 0 & 0 \\ 0 & 0 & 0 & + & - & - & 0 \\ 0 & 0 & + & 0 & 0 & - & 0 \\ 0 & 0 & 0 & + & 0 & 0 & - \end{bmatrix} \text{and, } \operatorname{sgn}\gamma = \begin{bmatrix} ? & ? & 0 & 0 & 0 & 0 & 0 \\ 0 & ? & ? & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ? & ? & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & + & - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ - & 0 & 0 & 0 & 0 & 0 & 0 \\ + & 0 & 0 & 0 & 0 & - & 0 \end{bmatrix}, \quad (7^*)$$

with the entries marked "?" assigned a positive or negative sign, with equal probably, the sign pattern of the outcome (8) is impossible, for the signs of the other entries in (7*) imposed. Whatever structural arrays are estimated, based upon (8), they could not possibly have resulted in this sign pattern for the estimated reduced form.

V. The Oil Market Simulation (OMS). The OMS (System's Science, Inc. (1985), EIA (1990)) was used in the 1990s by the Department of Energy's Energy Information Administration (EIA) in conjunction with a model of domestic energy markets in the preparation of annual long term forecasts and special studies as requested. The model's output is the world oil price (WOP), given estimates of supply and demand in some number of regions world-wide (seven in the version of the model used by EIA). Four other equations give the demand for OPEC production as needed to balance world-wide supply and demand; and, additionally express the relationship between the WOP in terms of OPEC's rate of capacity utilization. In general, the relationships used were nonlinear.

Remarkably, Hale and Lady (1995) conducted a qualitative analysis of the model as used by EIA and found that the corresponding reduced form was entirely signable. Further, given the generic nature of the regional supply and demand functions, the reduced form was signable regardless of the level of aggregation, i.e., it didn't matter for signability as to how many regions the world was divided up into. Lady and Buck (op. cit.) aggregated the model to contain only one, non-OPEC region, rendering the model one of six equations. In the format used there, and here, the structural form of the model is expressed by,

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \beta_{16} \\ 0 & 1 & 0 & 0 & 0 & \beta_{26} \\ \beta_{31} & \beta_{32} & 1 & 0 & 0 & 0 \\ 0 & 0 & \beta_{43} & 1 & 0 & 0 \\ 0 & 0 & 0 & \beta_{54} & 1 & 0 \\ 0 & 0 & 0 & 0 & \beta_{65} & 1 \end{bmatrix} \begin{bmatrix} dD \\ dS \\ dDO \\ dCAPUT \\ dR \\ dWOP \end{bmatrix} = \begin{bmatrix} \gamma_{11} & 0 & 0 & 0 \\ 0 & \gamma_{22} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_{43} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{64} \end{bmatrix} \begin{bmatrix} dD_{-1} \\ dS_{-1} \\ dMaxCap \\ dWOP_{-1} \end{bmatrix}.$$

Here, D is world oil demand, S is non-OPEC world oil supply, DO is the demand for OPEC oil, CAPUT is the rate of OPEC capacity utilization, R is the percentage change (in decimal) of the current WOP over last year's, MaxCap is maximum OPEC capacity and WOP is the world oil price. The sign patterns for the arrays are given below,

$$\operatorname{sgn} \beta = \begin{bmatrix} + & 0 & 0 & 0 & 0 & + \\ 0 & + & 0 & 0 & 0 & - \\ - & + & + & 0 & 0 & 0 \\ 0 & 0 & - & + & 0 & 0 \\ 0 & 0 & 0 & - & + & 0 \\ 0 & 0 & 0 & 0 & - & + \end{bmatrix}, and \ \operatorname{sgn} \gamma = \begin{bmatrix} + & 0 & 0 & 0 \\ 0 & + & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & - & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & + \end{bmatrix} \quad (9).$$

Following Hale and Lady's (op. cit.) qualitative methodology, Lady and Buck (op. cit.) showed that the reduced form for this aggregated version of the model <u>must</u> have the following sign pattern,

$$\operatorname{sgn} \pi = \begin{bmatrix} + & + & + & - \\ + & + & - & + \\ + & - & + & - \\ + & - & - & - \\ + & - & - & - \\ + & - & - & + \end{bmatrix} \quad (10).$$

To reiterate this result, for the sign pattern of the structural arrays given in (9); and, for $\pi = \beta^{-1}\gamma$, the sign pattern in (10) is the <u>only</u> possible sign pattern for the reduced form, regardless of the magnitudes of the entries of the structural arrays. Lady and Buck then estimated the reduced form based upon recent data with the following result,

OMS Estimated (1983-2006) Reduced Form Sign Pattern

	dD_{-1}	dS_{-1}	dMaxCap	$dWOP_{-1}$
dD	+	+	+	-
dS	-*	+	-	-*
dDO	+	+*	+	+*
dCaput	-*	-	+*	+*
dR	+	-	+*	-
dWOP	+	-	-	-*

("*" indicates a "wrong" sign compared to the predicted reduced form sign pattern)

There are nine entries of the estimated reduced form with different signs than those required by the sign pattern of the hypothesized structural arrays. So, in these terms, the hypothesized structure has been falsified.

Lady and Buck (op. cit.) did not go further and construct estimates of the structural arrays and it is beyond our scope to do so. The issue that we are raising here is whether or not, if the structural arrays had been estimated, the estimated structure could have possibly generated the data upon which the estimated reduced form is based. From our analysis, we found that the answer was "no." In particular, no arrays with the zero restrictions given in (9) could have led to the sign pattern in (10), regardless of the signs and values of the nonzeros. The sign pattern of the estimated reduced form falsifies the zero restrictions, independent of the nature of the nonzeros in (9).

This finding is based on using the Monte Carlo sampling procedure on the following arrays,

$$\operatorname{sgn} \beta = \begin{bmatrix} ? & 0 & 0 & 0 & 0 & ? \\ 0 & ? & 0 & 0 & 0 & ? \\ ? & ? & ? & 0 & 0 & 0 \\ 0 & 0 & ? & ? & 0 & 0 \\ 0 & 0 & 0 & ? & ? & 0 \\ 0 & 0 & 0 & 0 & ? & ? \end{bmatrix}, \textit{and} \operatorname{sgn} \gamma = \begin{bmatrix} ? & 0 & 0 & 0 \\ 0 & ? & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & ? & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ? \end{bmatrix} \quad (9^*).$$

As before, for the entries marked "?," the sampling procedure randomly selected positive or negative entries with equal probability. For the 6 x 4 reduced form, there are twenty-four bits needed to express the sign pattern ("1" for "+" and "0" for "-"); as a result, the Monte Carlo software could keep track of all of the different reduced form sign patterns found in the samples (tens of millions of sampled quantitative realizations of the structure (9*)). Altogether, there are 2^{24} = 16,777,216 possible 6 x 4 sign patterns that the estimated reduced form could take on (barring zeros). Of these, the Monte Carlo sampling procedure only found 4,099; and, significantly, none of these had the sign pattern found above for the estimated reduced form. Accordingly, the zero restrictions have been falsified. Given this, any estimated structural arrays with these zero restrictions imposed, based upon the estimated reduced form are impossible in that they could not possibly have generated the sign pattern found for the estimated reduced form.

VI. Impossibility Theorems and Conclusions. Implicit to all of our examples have been what might be termed "impossibility theorems." Take the first example, for the zero restrictions as expressed in (5*), 17,729 4 x 4 sign patterns were not found by the Monte Carlo sampling algorithm in tens of millions of samples, one of these is,

$$\operatorname{sgn} \pi = \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & + & + & - \\ - & + & - & + \end{bmatrix}.$$

Accordingly, the associated impossibility theorem would be:

$$\text{If } \gamma = I,\ \operatorname{sgn} \beta = \begin{bmatrix} ? & ? & 0 & ? \\ ? & ? & ? & 0 \\ 0 & ? & ? & ? \\ ? & 0 & ? & ? \end{bmatrix} \text{, and } \pi = \beta^{-1}\gamma, \text{ then } \operatorname{sgn} \pi = \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & + & + & - \\ - & + & - & + \end{bmatrix} \text{ is impossible}$$

for any nonzero entries in β corresponding to the entries marked "?."

Similar theorems are implied by our other examples, These findings are based upon the Monte Carlo sampling procedure's findings (or lack of findings). The decisive proof of such theorems would be a demonstration of an inconsistency in writing out the expansions of the terms in π as related to the proposed signs. In general, such demonstrations are beyond our scope. For sufficiently large samples for the Monte Carlo, the likelihood that such a demonstration cannot be made is vanishingly small; still, there is a likelihood, if the result is only based upon the Monte Carlo.

One way to proceed is to utilize the graph theoretic methods employed historically in investigating the signability of the entries in a matrix' inverse, based only upon the matrix' sign pattern, e.g., Hale, *et al* (1999). Consider the simplest inference structure that might be presented, the case of an irreducible matrix with only one inference cycle, say for the case n = 4,

$$\operatorname{sgn} \beta = \begin{bmatrix} ? & 0 & 0 & ? \\ ? & ? & 0 & 0 \\ 0 & ? & ? & 0 \\ 0 & 0 & ? & ? \end{bmatrix}.$$

As before, entries marked "?" are nonzero, but can be positive or negative. For this inference structure, there is only one path of inference between any two vertices of the corresponding signed directed graph. Given the algebra of writing out the expansions of each cofactor, e.g., Maybee (1966), since there are only eight (= 2n) nonzeros, results in only $2^8 = 256$ possible sign patterns for β's adjoint. Although when non-singular (as we always assume), the determinant can be either positive or negative this makes no difference, since the adjoint's sign patterns partition into two groups with one group the negative of the other. Hence, there are only 256 possible sign patterns for sgn β^{-1}. This compared to the 2^{16} = 65, 536 sign patterns, barring zeros, that are possible for a 4 x 4 array. Accordingly, the zero restrictions are falsified by the vast majority of possible reduced form sign patterns (assuming $\gamma = I$); and, should any of these be estimated, any estimates of the corresponding structural array would be impossible, if the zero restrictions are imposed. For this (admittedly very basic inference structure), for any value of n > 2, there are only 2^{2n} possible sign patterns for sgn β^{-1}, which is an ever increasingly small proportion of the 2^{nn} sign patterns that an n x n array might take on. Working with inference structures in this way may provide a promising route of derivation in showing the validity of the Monte Carlo's findings.

The basic point of the paper is this: Not only are qualitative methods far more robust as a tool than previously thought in assessing if the sign patterns of hypothesized structural arrays are falsified by the sign pattern of the estimated reduced form; but, the potential for falsification can be even more general. Namely, that the zero (and related) restrictions on the structural arrays can be falsified, independent of the sign pattern of the nonzero entries. When this takes place, any estimates of the structural arrays with the zero restrictions imposed is impossible. (Such as) the Monte Carlo procedures utilized for the results given in this paper, should become part of standard econometric practice to avoid such invalid derivations.

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Appendix

The Monte Carlo approach we utilize in this paper was first presented in Lady and Buck (2011). The Monte Carlo algorithm used here undertakes repeated sampling of the magnitudes of the entries of the structural arrays, consistent with the specified sign patterns. Then, $\pi = \beta^{-1}\gamma$ is computed. Given this, it is a simple procedure to count the number of times each entry of π is positive and the number of times negative (if zero the sample is discarded, since it is assumed that the structural arrays are such that the reduced form contains no entries that are logically zero). This is an extremely robust method to determine if any of the entries are signable, since if so they will always turn out to have the same sign, regardless of the size of the sample. Further, these simple counts will also reveal if entries of π always have the same, or different signs, as dictated by entries of β's adjoint being signable, since if so the counts of such will be the same, independent of the size of the sample. If the sign pattern of an estimated reduced form is in-hand, it is additionally straight-forward to see if the sign patterns of any of the sampled reduced forms are the same as the estimated reduced form. The size of the arrays is not particularly limiting for this “simple search,” except for the time needed to construct samples of a given size. If the sign pattern of the estimated reduced form is not found due to repeated sampling, then the counts of positive and negative entries in the sampled reduced form may reveal the reason, i.e., due to signable entries or entries required to always have the same or different signs. Typically (unfortunately), the sampled reduced forms will not provide these regularities and call for further analysis if the sign pattern of the estimated reduced form is not found, suggesting that the structural hypothesis is falsified.

Processing the algebra of the computation $\pi = \beta^{-1}\gamma$ is one of several obvious features of the analysis that calls for substantial development and algorithmic support. One immediate technique is to check to see if subgroupings of the reduced form sign pattern are resulting in the apparent falsification. This is accomplished by only checking such a subgrouping when comparing the sampled reduced forms to the estimated reduced form. When such subgroupings are identified, then the algebra of computing the reduced form can be written out with the appropriate focus to determine the problem of satisfying the proposed sign pattern. This was done in Lady and Buck (2011) and Buck and Lady (2012).

The method used here to enumerate all possible reduced form sign patterns is sensitive to the size of the system. For a given system, barring zeros as we assume, there are 2^{mn} possible reduced form sign patterns. The (long) integer used in our computing platform is limited to +/- 2^{31}. Accordingly, we cannot tabulate indexed counts of the reduced form sign patterns except for mn ≤ 30. This limitation can be mitigated by using other computing platforms or indexing schemes. The base 10 index associated with a reduced form sign pattern is computed from the binary number formed by writing out the array, row by row, with “1” corresponding to “+” and “0” corresponding to “-.” For example the index of

$$\operatorname{sgn} \pi = \begin{bmatrix} - & - & - & - \\ - & - & - & - \\ - & + & + & - \\ - & + & - & + \end{bmatrix}$$

presented in the last section of the paper as an impossible outcome for the zero restrictions presented in (5*) is “101.”