An Expanded Scope for Qualitative Economics

by

Andrew J. Buck

George M. Lady

Department of Economics

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1301 Cecil B. Moore Avenue, Philadelphia, PA 19122

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Abstract

As currently practiced, the analysis of an economic model's qualitative properties is very restricted and rarely productive. This paper provides an approach for conducting an expanded qualitative analysis that can be applied to any economic model. The method proposed will enable the qualitative properties of all economic models to be critically assessed.

Department of Economics

College of Liberal Arts

Temple University

Philadelphia, PA 19122

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1. Background. Samuelson (1947) proposed that economic theory expresses how the economy works via systems of simultaneous equations,

$$f^i(Y, Z) = 0,\ i = 1, 2, \ldots, n, \qquad (1)$$

where Y is an n-vector of endogenous variables and Z an m-vector of exogenous variables. Such systems are studied via the method of comparative statics when expressed by the linear system of differentials,

$$\sum_{j=1}^{n} \frac{\partial f^i}{\partial y_j} dy_j + \sum_{k=1}^{m} \frac{\partial f^i}{\partial z_k} dz_k = 0, i = 1, 2, \ldots, n. \quad (2)$$

In econometrics it is usual to assume that the system (2) is linear and, given this, that the system (2) can be expressed as,

$$\beta Y = \gamma Z + \delta U, \qquad (3)$$

where (β_{nxn}, γ_{nxm}, δ_{nxk}) are appropriately dimensioned arrays and U is a k-vector of the error terms. The system (3) is usually called the structural form and may be taken as the hypothesis of the theory. The classic papers by Klein (1950), Sims (1986), and Bernanke and Blinder (1992), and all the subsequent developments, are special cases of (3). The hypothesis (3) is brought to the data by estimating the entries of π in what is usually called the reduced form,

$$Y = \pi Z + V, \text{ for } \pi = \beta^{-1}\gamma. \qquad (4)$$

The estimate (4) can be used to "test" the theory, or following Popper (1959) potentially falsify[1] the theory, if it can be shown that the specification (3) imposes limits on the outcome for the estimated π. The theory would be falsified, subject to the statistical quality of the estimation, if the unrestricted estimate of π did not conform to the required limitations.

An immediate issue is what, just exactly, are the limitations on the outcome (4) as derived from the hypothesis (3). Samuelson noted that the issue was often made more difficult by the circumstance that the theory might only specify the sign patterns of (β, γ) in (3)[2]. Accordingly, the derivation of the limitations for the outcome of estimating π must be found by working with the sign patterns of the β and γ arrays. The enterprise of doing this is often termed a "qualitative analysis."

2. Qualitative Analysis. For ease of discussion, only consider hypotheses for which $\gamma = I$. This simplification is followed by most of the literature on qualitative analysis. An exception is Buck and Lady (2005), that does not limit the form of γ. Given this simplification, $\pi = \beta^{-1}$. The limitations upon π for a qualitative analysis are the signs of some, or all, of its entries. The hypothesis (3) is falsified, if the required signs do not appear in the estimated π. For this to be possible, there must be some way to work through the mathematics of inverting β, using only its sign pattern, sgn β, to come to the result of finding limitations on the sign pattern of π, independent of the magnitudes of the entries of β. Samuelson (op. cit.) did not believe that a successful qualitative analysis would be generally possible. Instead, he proposed a small number of other sources of limitation on the outcome of estimating π, e.g., that β is a stable matrix.

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[1] Falsification as distinct from statistical testing of hypotheses about the signs of the terms in β, γ, and δ.

[2] At the time that he wrote, Samuelson did not anticipate the developments of Barnanke and Blinder so we confine the discussion to β and γ.

A literature on the conditions on sgn β that enabled a successful qualitative analysis developed. Lancaster (1962) provided sufficient conditions for a successful analysis. These were generalized and algorithmic principles for conducting the analysis were developed, e.g., Bassett, *et al* (1968) and Lady (1983). A summary of much of this literature is given in Hale, *et al* (1999). None of this literature did much to dispel Samuelson's original doubts. Reported, successful qualitative analyses, e.g., Hale and Lady (1995), are extremely rare. The required conditions on sgn β are very restrictive and seldom satisfied. In general, there is no tradition of testing, or potentially falsifying, economic models using a qualitative analysis.

3. An Expanded Qualitative Analysis. The conditions so far developed in the literature for a successful qualitative analysis are too restrictive and do not account for very substantial opportunities to impose restrictions on sgn π based upon sgn β. A principle stumbling block in current qualitative methods is the requirement that β be shown to be nonsingular based only upon its sign pattern. This condition is almost never satisfied. We relax it and consider β without this restriction, assuming otherwise that β is nonsingular. This provides no significant limit on the systems we can study, since actual, applied models are virtually never singular. Additionally, we will assume that β is irreducible, i.e., that no entry of π must be zero. Given this, we find that the specification of <u>any</u> sign pattern for β provides some kind of restriction upon the signs that can be taken on by the entries of π. Even though no particular entry of π can be signed, it can be shown that:

Proposition: There are always certain sign patterns for entire rows, or columns, of π that are impossible, given the hypothesized sgn β.

Proof: This can be immediately seen by considering that $\beta\pi = I$. As a result (say), the terms being summed for the product of the kth row of β and the kth column π must contain at least one positive term, since the sum of terms will be "1." For the given sign pattern of the kth row of β, this requires that the kth column of π not contain all entries of the opposite sign from the corresponding nonzeros in the kth row of β, since for this all terms in the corresponding sum would be negative. By the same reasoning, the terms being summed for the product of the kth row of β and a column other than the kth column of π must contain at least one positive and at least one negative term, since the sum of the terms must be zero. For the kth row of sgn β given, this requires that any column of π other than the kth not contain all entries of the opposite sign from, or the same sign of, the corresponding nonzeros in the kth row of β, since for this all of the terms in the corresponding sum would be all negative or all positive. Since also, $\pi\beta = I$, similar limitations can be imposed on the signs of the entries of the rows of π, based upon the given signs in the columns of β.[3]

4. Summary. The point of these remarks is that any hypothesized sgn β could be potentially falsified, if any of the forbidden sign patterns for the rows or columns of π show up in the estimated π, regardless of what one might have concluded on the basis of the derived asymptotic t-statistics for the derived estimates of β. Further, the limitations on π's sign pattern due to a fully specified sgn β are often, perhaps always, far greater than the one's identified above. Buck and Lady (2010) have used a Monte Carlo algorithm to investigate the sign patterns found for π corresponding to a β with a given sign pattern and liberal rules for choosing the magnitudes of its non-zeros. As an example, we assessed the sign patterns of the inverses of a 4 x 4 version of β

[3] We initially developed these conditions utilizing Lancaster's (1966) elimination algorithm. In reviewing the conditions, James Quirk (Cal. Tech., retired) pointed out that the above argument was more immediate and transparent.

with an all negative main diagonal and all positive off diagonal terms (a so-called Metzler (1945) matrix). Assuming further that π has no zero entries, there are 65536 conceivably possible 4 x 4 sign patterns that π might take on. Of the sixteen possible sign patterns for each row/column of π, the above arguments show that seven sign patterns are impossible for each row/column. Based on this, there are only 6561 possible sign patterns for π, if the sign patterns are independent of each other. Using the Monte Carlo for tens of millions of trials, we only found 205 distinct sign patterns for π. This suggests that the sign patterns are not independent and further limitations exist for row/column sign patterns that, although individually possible, cannot appear in various combinations.

It is clear that a qualitative analysis can be applied to any model limited in its specification to the sign patterns, i.e., directions of influence among its variables, of its structural arrays. Considerable analytical and algorithmic development remains to be done to exploit the potential for submitting qualitatively specified economic models to analyses of their implication for reduced form outcomes. There really is no commonly agreed to protocol now in place to address the veracity of structural hypotheses that are often, at least in part, only expressed in terms of their qualitative content. The expanded qualitative analysis that we outline here is a good starting point for establishing a tradition of such critical assessments.

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