Funding Liquidity and Market Liquidity

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Abstract

Recent empirical studies have shown an increasing co-movement between fund and market liquidity, which is driven by common factors such as monetary shocks. Modeling this co-movement becomes desirable to evaluate policies relating to liquidity and financial instability. This paper establishes a monetary model with capital to explain the dynamic interactions between funding and market liquidity in a search framework featured by Kiyotaki and Wright [1989]. Capital and money are two important elements here. As the collateral and production input, capital affects both fund and goods trading market. As medium of exchange, money is essential to trade; meanwhile the opportunity cost of carrying it affects the fund market imbalance as well. As a result, monetary policy can change traders' expectations and negotiations, and have non-trivial impact on fund markets and liquidity risks. Calibrated the model, simulated liquidity moments respond to monetary shocks, moving together across time and presenting business cycle properties.

Key Words: Liquidity, Monetary Policy, Search and Matching

JEL code: E5, E51, E52, G12

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1 Introduction

In recent decades, financial crises are often triggered by liquidity shocks and accompanied with failures of the liquidity risk management. Although the cruciality of liquidity risks have been well recognized that they impact asset prices, trading volume and frequency, and predictions of future returns on financial assets. It's not until recent years that more attentions have been drawn to the interactions of liquidity risks across markets. The financial system would become more fragile if the co-movement between funding and market liquidity is stronger. In order to explain the empirical findings of the interaction among funding, market liquidity, and monetary shocks, this paper adopts Kiyotaki and Moore [1997] and Kiyotaki and Wright [1989] to model the fund and goods trading market, internalizing this co-movement.

It is a stylized fact that funding and market liquidity covary. Recent empirical works examine the time series of liquidity across markets, and document the commonality between liquidity and trading frictions. For example, Fleming et al. [1998] measures liquidity by the volatility of return and shows a strong linkage between funding and market liquidity. During the liquidity crisis, observed funding and market liquidity mutually reinforce one another. A small negative shock to the economy might be amplified through this mechanism and result in a sudden drying-up of the liquidity.

During the financial crisis, policy interventions are expected to alleviate the liquidity crunch. Chordia [2005] shows that the co-movement of liquidity across bond and asset market is driven by common factors such as monetary shocks. Shin et al. [2010] documents significant impacts of monetary policies on financial markets and financial stability. Adrian and Shin [2008] shows that monetary policy has a direct impact on broker-dealer asset growth via short-term interest rates, yield spread and risk measures. Piazzesi [2002] finds that an unexpected increase of federal funds rate will increase the transaction cost and thereby the trading friction of the stock market, hence lowering market liquidity, and *vice versa*. Although monetary policy does not usually target financial markets directly, it affects liquidity by changing transaction costs, trading activities, and *etc*.

Major literature on liquidity has developed separately on funding liquidity and market liquidity to answer policy related questions. Bernanke and Gertler [1989] analyzes how balance sheet liquidity affects output dynamics and thereby business cycles [1]. Kyiotaki and Moore's seminal work Kiyotaki and Moore [1997] shows persistent and amplified effects of shocks due to the dynamic interaction between credit limits and asset prices as a transmission mechanism. Thereafter, many have adopted their idea to study financial market frictions and the business cycle. Brunnermeier and Pedersen [2009] develops a theoretical framework to link funding and market liquidity. They provide inspiring explanations to the co-movement feature of liquidity risk. But their results are

[1] Bearing the same economic intuition, liquidity is a catch-all term that may refer to different concepts, for example, balance sheet liquidity or accounting liquidity. Price spreads, volatilities of return and market depths are frequently used as measures of liquidity.

restricted to binding equilibria, since trading is assumed to be competitive and frictionless that the matching of buyers and sellers is instantaneous and costless.

With this concern, a new body of research builds on micro-foundations to interpret trading frictions and the role of money explicitly. This effort was pioneered by Kiyotaki and Wright [1989, 1993] in monetary theory and by Duffie et al. [2005, 2007] in finance. Under this framework, Rocheteau and Weill [2011] and Rocheteau and Wright [2013] study asset pricing, market liquidity and monetary policies. This paper adopts this framework, proposing a search-based model with fiat money and endogenous borrowing constrains. The fund market is facilitated by secured loans since borrowers have limited commitment. The borrowing constraints of secured loans depend on the collateral and its pledge-ability, which is measured by endogenous loan-to-value (LTV) ratios. Similar to Kiyotaki and Wright [1989], the goods trading market employs fiat money as medium of exchange and subjects to search and matching frictions. Then market liquidity depends on the trading frequency and the ease of traders' negotiations; funding liquidity is measured by the imbalance of the fund market.

Firms are borrowers of secured loans, as well as sellers and producers of the goods market. If the fund market is more liquid, firms could invest more in capital, produce more efficiently and negotiate harder to trade, which implies an increased market liquidity. A more liquid goods trading market increases firms' profitability and hence pledge-ability of the capital, which could lead to a continuing cycle of increased funding liquidity. Capital is a key factor to the co-movement between funding and market liquidity. Similar to Kiyotaki and Moore [1997], firms' capital is not only the input of the production, but also the collateral asset for loans. More capital not only reduces production cost, but also decreases firms' thread point in the bargaining process. This argument is similar to Lagos and Rocheteau [2009] on liquidity of over-the-counter (OTC) markets. They argue that the exchanges of assets in OTC markets are not only affected by the current value of the asset, but also the holding cost of the asset for a certain period of time. Making an analogous, firms' capital and households' money are costly to hold over time and hence entering the bargaining process of the goods market. In the goods trading market, firms(sellers) and households(buyers) randomly meet each other, which is time consuming and that creates trading frictions. In a bilateral meeting, the buyer bargains with the seller to negotiate the price, and then trade fiat money for goods. Financial intermediaries set the LTV ratio to hedge default risks. On the equilibrium path, the optimal LTV ratios should be incentive compatible with no default of borrowers. This specification emphasizes the role of money in the exchange process, establishing a theoretical link between monetary policy and fund flows.

The conventional monetary policy targeting the interest rate i is modeled by a lump-sum money injection to the economy every period. A comparative static analysis shows that inflation hurts the intensive margin, in that the trading volume of the goods market decreases and the borrowing margin increases. On the other side, inflation may bring a trading opportunity effect, that an increased expected opportunity cost of holding money would increase households' probability of finding a successful match [2]. Whether inflation hurts or boosts liquidity depends on which effect dominants. If the economy has a mild inflation and households have very small bargaining powers over prices, the trading opportunity effect may dominate, and liquidity would be improved in the long-run with an increased participation of households relative to firms.

In the short-run, money injection channels matter. Injection to households may boost the market liquidity temporarily. Injection to firms instead of households increases the funding liquidity in constrained equilibrium, but decreases the funding liquidity in unconstrained equilibrium. Similar to the qualitative and quantitative theory of money in Samuelson [1968], monetary shocks on constrained and unconstrained equilibrium have bifurcate outcomes. In constrained equilibrium, the quantity of money is essential; while in unconstrained equilibrium, money is valued as the lubricant. Without an appropriate justification of economic parameters and equilibrium regimes, the monetary policy may lead to unwanted effect, such as increased illiquidity and inefficient production.

Pushing further, I calibrate the model quantitatively. The simulated market liquidity and funding liquidity move together across time in both constrained and unconstrained equilibrium. Liquidity also presents business cycle property. The rest of the paper is organized as follows. Section 2 sets up the model. Section 3 describes the monetary equilibrium with a bargaining solution. Section 4 calibrates the model; describes the co-movement and response of liquidity to monetary shocks numerically. Section 5 discuss the long-run and short-run effects of monetary interventions. Section 6 introduces extended models with price posting mechanism. Section 7 concludes.

2 Model

2.1 Environment

The economy is populated by households, firms, financial intermediaries and the Monetary Authority. Time is discrete and infinite.Households and firms are continuum-measure with mass unity exogenously given. Households are risk averse and discount the future by β, while firms are risk neutral and discount the future by $\frac{1}{R}$. Each period, a centralized market (CM) appears first, and then a decentralized market (DM) with trading frictions. In the first sub-period, firms borrow from financial intermediaries to make investment decision and produce; households work to earn labor income and then consume. The goods produced in the first sub-period are non-storable general goods that both households and firms can consume, abbreviated as CM goods. CM goods can be transformed into investment goods one-for-one to build up the capital stock. Firm's capital dynamic follows the setting of the neoclassical growth model. In the following sub-period, firms may produce if they meet and trade with households. The goods produced in this sub-period (DM

[2]Trading opportunity effect is well defined in Shi [1999]

goods) is non-storable special goods that only households can consume. The DM goods production is contingent on the bilateral meetings between firms and households. The meeting probability for the household is α_f, and α_h for the firm. Once matched, firms produce DM goods y and households pay d for it. Here fiat money is the only means of payment in DM goods trades, which implies that money is universally valued and accepted. The non-storable property of CM and DM goods also supports the essentiality of money [3]. Briefly speaking, firms invest in the first sub-period only, but may produce in both sub-periods; households earn labor income in the first sub-period only, but may consume in both sub-periods. Hence, firms and households have liquidity needs at different timing, first and second sub-period respectively, that liquidity intermediation is desirable.

Financial intermediaries are risk neutral, with full commitment and enforcement. On one side, financial intermediaries issue risk-free bonds with gross interest rate R^f; on the other side, they lend out secured loans with gross interest rate R [4]. The intermediation takes place in every first sub-period. Since borrowers can not commit on loan payments, financial intermediaries impose borrowing limits to reduce the default risk. Here the borrowing constraints imposed by financial intermediaries are internalized in the fashion of Kiyotaki and Moore [1997], that borrowers are required to provide certain amount of collaterals, capital k, to secure loans. On the event of default, financial intermediaries could seize the collateral to compensate their losses and punish defaulters.

Finally, the monetary authority injects liquidity into the economy. The monetary policy is modeled as a lump sum transfer τ from the monetary authority to households at the beginning of first sub-period. In order to target a money gross growth rate ν to induce a specific interest rate, the amount of money transfer depends on the aggregate money supply M^s from the last period, that $\tau = M^s_{t-1}(\nu - 1)$.

2.2 Households

Households are assumed to live forever, which excludes age as an explicit state argument. They have quasi-linear utility, $U(x) - AL + \alpha_h u(y)$, where $U(x)$ is the utility of consuming CM goods x, $u(y)$ is the utility of consuming DM goods y and AL is the disutility of work. Both $U(x)$ and $u(y)$ are increasing and strictly concave. AL is linear in working hours L. Let CM goods be the numeraire. The household's inter-temporal budget constraint is

$$\phi(m' - m - \tau) + x = wL - \frac{b'}{R^f} + b \tag{1}$$

$$y \leq m'$$

[3] The assumptions of money and search frictions are standard in money search literature. Some papers also allow a mixture of financial assets and money as payment in trades, which naturally raises the issue of asset pricing. Without loss of generality, this paper only allows money as means of payment to focus on the searching and matching frictions of the second sub-period.

[4] To focus on interior solutions, set $R^f = R$

where w is the real wage rate, ϕ is the price of fiat money in the current period, m is the money holding at the beginning of the current period and $m^{'}$ is the amount of money brought to the second sub-period by household to buy DM goods y. Household invests on the risk free bond issued by financial intermediaries with discount price $\frac{1}{R^f}$. In the budget constraint (2), b is the current bond holding and $b^{'}$ is the next period's bond holding. In the second sub-period, households have chances to meet pairwise with firms to trade bilaterally. Let $n = \frac{N_h}{N_f}$ denote the participation ratio of households over firms. Normalize $N_f = 1$. Then the meeting probability, α_f, is increasing in the market intensity n of the second sub-period. Follow Shi [2006], assume the matching function $\alpha(n)$ is constant return to scale, that $\alpha^{'}(n) > 0$, $\alpha^{''}(n) < 0$, $\alpha(n) \leq min\{1, n\}$, $\alpha^{'}(0) = 1$ and $\alpha(\infty) = 1$. The meeting probabilities for households and firms are $\alpha_h = \frac{\alpha(n)}{n}$ and $\alpha_f = \alpha(n)$ accordingly. Since money is the only medium of exchange in the second sub-period, whether to hold more money $m^{'}$ or bond $b^{'}$ depends on the tradeoff between consuming in the subsequent period or next period.

2.3 Firms

Firms are risk neutral and maximize profits. The CM goods production is $f(k, L)$ where k is capital input and L is labor input. The DM goods production only require capital input. The cost function of producing y DM goods with capital $k^{'}$ is $c(y, k^{'})$, which satisfies the following properties $c_1(y, k^{'}) > 0$, $c_2(y, k^{'}) < 0$, $c_{11}(y, k^{'}) > 0$, $c_{22}(y, k^{'}) > 0$ and $c_{12}(y, k^{'}) < 0$ [5]. Similar to Kiyotaki and Wright [1989] and other related money search literature, DM trade is bilateral and *quid pro quo*; therefore the price of DM goods may contain a bubble which leads to inefficient production.

The dynamic of capital accumulation is settled in the first sub-period. The capital k depreciates by δ after CM goods production. Firms decide the investment I and hence the capital stock in the second sub-period, $k^{'}$, that

$$k^{'} = (1 - \delta)\, k + I^{6}. \tag{2}$$

Assume firms make capital investment at the beginning of every period which creates liquidity needs. To produce more, they would like to borrow from financial intermediaries. This assumption on timing is not just convenient to the theory, but also realistic since productions are usually not instantaneous. Besides, assume firms can not commit on the debt, that they need to use collateral to secure the loan. Lender would impose the following borrowing constraint depending on the amount of the collateral $k^{'}$ and the loan-to-value ratio (LTV), $\gamma^{'}$, that

$$b^{'} \leq \gamma^{'} k^{'} \tag{3}$$

Firms take $\gamma^{'}$ as given. Financial intermediaries set the LTV ratio, $\gamma^{'}$, to be incentive compatible

[5] I adopt the cost function setup from Aruoba et al. [2009]. The cost function is strictly increasing and convex in y and strictly decreasing and convex in $k^{'}$. Moreover, the cross term $c_{12} < 0$ guarantees that more capital always increases DM production.

with no defaulting on loans. Firm's inter-temporal budget constraint in the first sub-period is

$$z + I = f\left(k, L\right) - wL + \frac{b^{'}}{R} - b \tag{4}$$

where z is the net profit, $\frac{b^{'}}{R}$ is the fund inflow from the new loan and b is loan position at the beginning of every period. Here $b^{'}$ is the loan rolled to the second sub-period, it might be different from the loan position at the beginning of next period. If firms could sell DM goods and earn d, the loan $b^{'}$ would be reduced to $b^{'} - d$.

2.4 Planners

As a benchmark, consider the planner's problem with perfect credit. In each period, the planner chooses general goods production $f\left(K, L\right)$ and special goods production y. The planner also decides allocation of the production between households and firms, as well as the capital accumulation dynamic. Assume the planner can not avoid the search and matching friction, that firms are matched with households randomly in the second sub-period. Households consume y with probability α_h and firms earn revenue d with probability α_f.

The planner's goal is to maximize the sum of households and firms' utilities subject to the resource constraint (6) and the capital dynamic equation (7) [7].

$$\begin{aligned} H(k) &= \max_{x,y,z,L,I,n} \quad n\left[U(x) - AL\right] + z + \alpha(n)\left[u(y) - c(y, k^{'})\right] + \beta H(k^{'}) && (5)\\ & s.t. \qquad x + z + I = f\left(k, L\right) && (6)\\ & \qquad\qquad I = k^{'} - \left(1 - \delta\right)k && (7) \end{aligned}$$

Substitute (6) and (7) to (5), it is straightforward to show that the planner's problem has interior solution and full participation is socially optimal. Given $\tilde{n} = 1$, other optimality conditions are as follows:

$$U^{'}\left(\tilde{x}\right) = 1 \tag{8}$$

$$u^{'}\left(\tilde{y}\right) - c_1\left(\tilde{y}, \tilde{k}^{'}\right) = 0 \tag{9}$$

$$-\alpha(\tilde{n})c_2(\tilde{y}, \tilde{k}^{'}) + \tfrac{1}{R}f_1(\tilde{k}^{'}, \tilde{L}) = 1 - \beta(1 - \delta) \tag{10}$$

$$f_2(k, \tilde{L}) = A \tag{11}$$

and $\tilde{z} = f\left(k, L\right) - I - x \geq 0$. Here planner arranges credit and participation. Productions in both sub-periods are socially optimal. In the decentralized economy, the credit limit and trade frictions would cause efficiency loss that $k^{'} < \tilde{k}^{'}$ and $y < \tilde{y}$.

[7] The planner's problem can be written as the maximization of households' utilities given firms' making even because households have quasi-linear utility and firms have linear utility.

2.5 Decentralized Economy

In the decentralized economy, households and firms optimize with borrowing constraint and trade frictions. Let $W^H(m, b)$ be the household's optimal continuation value at the beginning of every period, and $V^H(m', b')$ at the beginning of the sub-sequent second sub-period. The household brings money m into each period, then adjusts it to m' for potential DM goods consumption. Another argument b is the bond holding at the beginning of each period. The households adjusts the bond holding to b' in the first sub-period, and then bring it to next period. In the first sub-period, households solve the following dynamic problem subject to the inter-temporal budget constraint (2)

$$W^H(m, b) = \max_{x, L, b', m'} U(x) - AL + V^H(m', b') \tag{12}$$

In the second sub-period, household's continuation value depends on the terms of trade $\{d, y\}$, that

$$\begin{aligned} V^H\left(m', b'\right) &= \alpha_h \left[u(y) + \beta W^H(m' - \frac{d}{\phi'}, b')\right] \\ &\quad + (1 - \alpha_h)\,\beta W^H\left(m', b'\right) \end{aligned} \tag{13}$$

Substitute L in (12) with the the one calculated from (2). Envelope conditions, $W_1^H(m, b) = \frac{A\phi}{w}$ and $W_2^H(m, b) = \frac{A}{w}$, imply linearity of W on m and b. Hence (13) can be simplified to

$$V^H(m', b') = \beta W^H(m', b') + \alpha_h \left[u(y) - \beta \frac{A}{w'} d\right] \tag{14}$$

where $\left[u\left(y\right) - \beta \frac{A}{w'} d\right]$ is the household's gain of trade in the second sub-period. The exchange of money d and DM goods y is resolved by the Nash bargaining process, which depends on the negotiation between households and firms.

The firm makes production, investment and finance decisions according to the debt position, capital and an exogenous LTV ratio. Firms' optimal continuation values are defined as $W\left(b, k\right)$ at the beginning of every period, and $V\left(b', k'\right)$ at the beginning of the second sub-period. Here b is firms' debt rollover from the last period and b' is the new debt position. The debt rolled to the next period would be $b' - d$ if firms produce and trade in the second sub-period, b' otherwise. k is the capital stock at the beginning of every period. Only in the first sub-period, firms can invest in capital goods and increase capital stock from k to k'. Every period, firms solve the following dynamic problem (16) subject to the budget constraint (4), capital dynamic (2) and borrowing constraint (3).

$$
\begin{aligned}
W(b,k) &= \max_{z\geq 0, I\geq 0, L, b'} z + V\left(b', k'\right) && (15)\\
V\left(b', k'\right) &= \alpha_f\left[-c\left(y, k'\right) + \frac{1}{R}W\left(b' - d, k'\right)\right] && (16)\\
&\quad + (1-\alpha_f)\frac{1}{R}W\left(b', k'\right)
\end{aligned}
$$

Substitute z using (4), I using (2) and rewrite firms' problem (16) as

$$
\begin{aligned}
W(b,k) &= \max_{L,b',k',\lambda} f(k,L) - wL + \tfrac{b'}{R^f} - b - k' + (1-\delta)k + V(b',k') + \lambda(\gamma' k' - b')\\
&= -b + (1-\delta)k + \max_{L,b',k',\lambda}\left\{f(k,L) - wL + \tfrac{b'}{R^f} - k' + \lambda(\gamma' k' - b') + V(b',k')\right\}
\end{aligned}
$$

where λ is the Lagrange multiplier for constraint (3). Then the envelope condition implies that $W_1(b,k) = -1$. The second sub-period problem (16) can be simplified as

$$
V\left(b', k'\right) = \frac{1}{R}W\left(b', k'\right) + \alpha_f\left[-c\left(y, k'\right) + \frac{1}{R}d\right] \qquad (17)
$$

where $\left[-c\left(y, k'\right) + \frac{1}{R}d\right]$ is firm's gain of trade.

With rational expectations of household's and firm's behavior, risk neutral financial intermediaries make zero profit and set the LTV ratio γ' to prevent default. Assume firms lose all the collateral if they default. They can not choose to default on part of the debt. If firms invest one more unit of capital, they can borrow $\gamma'\beta$ from the financial intermediaries. The borrowing margin, or haircut in finance, is then $1-\gamma'\beta$, which is the marginal default cost for firms. The marginal gain of default is one-period marginal revenue from the extra unit investment, which is decreasing in k'. So it's sufficient for financial intermediaries to choose the γ' equalizing the marginal default cost and gain, that

$$
\underbrace{1-\gamma'\beta}_{Marginal\ Default\ Cost} = \underbrace{-\alpha_f c_2\left(y, \hat{k}'\right) + \frac{1}{R}f_1\left(\hat{k}', L'\right)}_{Marginal\ Default\ Gain} \qquad (18)
$$

where $\hat{k}'$ represents the optimal capital stock. If γ' is larger than the equilibrium, firms may default since the option value of default becomes positive due to a thin margin. As a result, financial intermediaries would decrease γ' to tighten the credit. If γ' is smaller than optimal, firms would under-invest due to the credit limit, which drives up the price level. Then, financial intermediaries would adjust the pledge-ability of firms' collateral and loosen the credit limit to extend lending.

2.6 Nash Bargaining

In randomly assigned pairwise meetings between households and firms, the terms of DM goods trades, d and y, are resolved by a standard Nash Bargaining process. Let θ be the households' bargaining power and $(1-\theta)$ be the firms' bargaining power. According to (14) and (17), households' gain of trade is $\frac{A}{w'}\left[\frac{w'}{A}u(y)-\beta d\right]$, and firms' gain of trade is $\left[-c(y,k')+\frac{1}{R}d\right]$. Then (d,y) solve the following problem:

$$\begin{aligned} &\max_{y,d} \quad \left[\frac{w'}{A}u(y)-\beta d\right]^{\theta}\left[-c(y,k')+\frac{1}{R}d\right]^{1-\theta} \\ &s.t. \qquad\qquad d \leq \phi' m' \end{aligned} \tag{19}$$

Here multiplier $\frac{A}{w'}$ is ignored from households' gain of trade to make the solution concise. Since $\frac{A}{w'}$ is exogenous to the choice variables y and d, it does not affect the optimization result. In equilibrium, $R\beta = 1$. So the total gain from trade on the equilibrium path is $\frac{w'}{A}u(y) - c(y,k')$, which is non-negative if the bargaining solution exists [8].

Define y^* as the Pareto optimal/efficient DM goods production, and it solves $\frac{w'}{A}u'(y^*) - c_1(y^*,k') = 0$. Let d^* be the corresponding payment for y^*, and $m^* = d^*/\phi'$. Since the efficient DM goods production y^* does not depend on the price, d^* and m^* can be solved by maximizing the bargaining problem (19) given $y = y^*$, that

$$\beta d^* = \beta\phi' m^* = \theta c(y^*,k') + (1-\theta)\frac{w'}{A}u(y^*) \tag{20}$$

If $m' \geq m^*$, $d = \phi' m^*$ and $y = y^*$; if $m' < m^*$, $d = \phi' m'$ and y solves $d = g\left(y,k'\right)$, where

$$g\left(y,k'\right) \triangleq \frac{\theta u'(y)c(y,k') + (1-\theta)c_1(y,k')u(y)}{\theta\frac{1}{R}u'(y) + (1-\theta)\frac{\beta A}{w'}c_1(y,k')}. \tag{21}$$

If households bring sufficient money to the second sub-period ($m' \geq m^*$), the DM goods production would be efficient. However, the Nash bargaining mechanism can not induce the first-best solution with money because of the potential holdup problem for both households and firms. The meeting of DM goods trade is random and hence there is always positive probability that households couldn't spend their money or firms couldn't produce. Lagos and Wright [2005] proves that $m' < m^*$ in a monetary equilibrium since the expected utility from participating the second sub-period would be negative if $y = y^*$. If $m' < m^*$, it is optimal for households to spend all the money they brought to the second sub-period. First, holding money is costly; second, households' DM goods consumption y is strictly increasing in payment d [9].

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[8] The discussion with a negative total gain, $\frac{w'}{A}u(y) - c(y,k') < 0$, is not of the interest here. One can impose assumptions on the utility and cost function parameters to avoid it.

[9] $y'_d = \frac{1}{g_1\left(y,k'\right)} > 0$ if $m' < m^*$ since the utility function $u\left(y\right)$ is strictly concave, and the cost function $c\left(y,k'\right)$

Proposition 2.1 *Terms of trade* (d, y) *solve the Nash bargaining problem* (19) *if* $d = \phi' m'$ *and* y *solves* $\beta d = g\left(y, k'\right)$ *where* $g\left(y, k'\right)$ *is defined in* (21).

Notice that firm's capital stock k' is essential to the allocation of the total surplus from trade. A larger k' decrease firm's share of surplus, $\left[g\left(y, k'\right) - c\left(y, k'\right)\right] / \left[\frac{w'}{A} u\left(y\right) - c\left(y, k'\right)\right]$. Intuitively, the firm needs to borrow more in the first sub-period to build up a larger capital stock, which increases the hold-up cost of its debt. Therefore firm has stronger incentive to offer a buyer-friendly price to secure the trade. This argument is similar to Lagos and Rocheteau [2009], which emphasizes the holding cost rather than the production cost to understand the role of k'. Meanwhile the total gain from the trade, $\frac{w'}{A} u\left(y\right) - c\left(y, k'\right)$, is increasing in k'. Hence the optimal investment decision balances the tradeoff between a larger total surplus and smaller share.

3 Funding and Market Liquidity

3.1 Equilibrium

If the total trade surplus of DM goods is non-negative, firms always participate. Households would consider the expected utility of participating the second sub-period to decide whether to bring positive amount of money m' or not. If participation yields negative utility, households would deposit all the available fund to financial intermediaries instead.

Corollary 3.1 *In equilibrium, the* **participation** *ratio of households to firms,* n*, is such that households' expected utility of participation is greater or equal to zero,*

$$\left\{-i\frac{A}{w'}g(y, k') + \alpha_h\left(n\right)\left[u(y) - \frac{A}{w'}g(y, k')\right]\right\} \quad \geq \quad 0. \tag{22}$$

n *is binding if* (22) *is not binding; if* (22) *is binding, households are indifferent between participating and not, and* n *is determined by the following equation*

$$\alpha_h\left(n\right) = \frac{i\frac{A}{w'}g(y, k')}{u(y) - \frac{A}{w'}g(y, k')}. \tag{23}$$

Then the *equilibrium* is defined as follows

Definition 3.1 *Given households' money and bond holding* $\{m, b\}$*, firms' debt and capital stock* $\{b, k\}$ *and current price of money* ϕ*, the equilibrium satisfies the following conditions all together:*

(1) given $\{d, y\}$*,* γ'*,* n*,* w *and* ϕ'*, households' choice variables* $\left\{L, x, m', b'\right\}$ *solve* (12)*, firms' choices variables* $\left\{L, z, k', b'\right\}$ *solve* (16)*;*

is strictly increasing and convex in y and strictly decreasing and convex in k'. See appendix for further proof.

(2) given condition (1) satisfied, $\{d, y\}$ *solve the bargaining problem* (19)*;*

(3) given (1) and (2) satisfied, γ' *solves* (18)*;*

(4) ϕ' *clears the CM goods market*

$$\int_h x + \int_f z = \int_f [f(k, L) - I]; \tag{24}$$

(5) bond market clears $\int_h b' = \int_f b'$*; and* w *clears the labor market,* $\int_h L = \int_f L$

(6) participation ratio n *is determined by Corollary 3.1*

CM goods consumption x would achieve interior solution if $U'(x) = \frac{A}{w}$. In a very general case, corner solutions of x is possible but trivial to the liquidity risks discussed in this paper. To avoid the complexity of corner solutions, choose the functional form of U such that $\lim_{x \to 0} U'(x) = \infty$ and $U'(x)$ decreases fast enough with regard to x. Then the optimal x always satisfies the first order condition. The marginal utility of b' is linear, such that, $b' = 0$ if $w' > w$; $b' > 0$ if $w' \leq w$. The first order condition of m' implies that $m' > 0$ if

$$\phi' \frac{A}{w'} \left\{ \alpha_h \beta \left[\frac{u_y(y)}{g_1(y, k')} - 1 \right] - \frac{w'}{w} \frac{\phi}{\phi'} + \beta \right\} \geq 0, \tag{25}$$

and equality of (25) achieved at interior solution of m'. Using the above results, we can briefly discuss three regions of (b', m'): no money equilibrium with debt; monetary equilibrium with and without debt b'. If (25) is not satisfied, $m' = 0$, that the DM goods market would be shut down since it is too costly to carry money. This may happen if the inflation, $\frac{\phi}{\phi'}$, is significantly high, or the matching between households and firms is too frictional that α_h is very small for any participation ratio n. The no money equilibrium also implies zero market liquidity and minimum funding liquidity. In a monetary equilibrium without debt, funding liquidity is zero, $b' = 0$, and market liquidity is at the minimum. Therefore the investment is shrinking on the equilibrium path and the economy would eventually collapse. Finally, in the equilibrium with both money and debt, households are indifferent between holding bond and money. Then the optimality conditions of b' and m' are $w = w'$ and

$$\frac{\phi}{\beta \phi'} \frac{w'}{w} - 1 = \alpha_h \left[\frac{w'}{A} \frac{u'(y)}{g_1(y, k)} - 1 \right]. \tag{26}$$

The left hand side of (26) is the opportunity cost of carrying one unit of money into the second sub-period, and the right hand side is the expected marginal returning of money. Both market and funding liquidity are positive in this regime. In order to induce a positive money holding in equilibrium, we need to put certain restrictions on the curvature and position of the utility function $u(y)$ and cost function $c(y, k')$.

Assumption 3.1 $\frac{u'(y)}{g_y(y,k)}$ *is strictly decreasing in* y $\forall k$*, and* $\lim_{y\to 0}\frac{u'(y)}{g_y(y,k)} = \infty$.

With the above assumption, a monetary equilibrium would exists if households always expect positive return from participating the second sub-period.

Proposition 3.1 *A Monetary Equilibrium exists if and only if*

$$\max_y \left\{-i\frac{A}{w'}g(y,k') + \alpha_h\left[u(y) - \frac{A}{w'}g(y,k')\right]\right\} > 0 \tag{27}$$

where k' *is optimal for firms.*

For a rigorous analysis of the co-movement between funding and market liquidity, the rest of this paper will focus on the monetary equilibrium with positive debt. Then households are indifferent between working more and holding more bonds. Households' bond holding b' and labor supply L would be pinned down together by (2) and market clear for labor. The optimality condition of firms' labor demand is $f_2\left(k', L\right) = w$, debt demand $b' = \gamma' k'$, and $\lambda = 0$. Interior solution of k' satisfies

$$1 - (1-\delta)\beta = \beta f_1(k', L') - \alpha_f(n)c_2(y, k') \tag{28}$$

In order to finance k', the firm needs to borrow $b^I \triangleq I - f(k,L) + wL + b$. Let the solution to (28) be k^u, which is only feasible if $b^{I(k^u)}$ is within the borrowing limit that $b^{I(k^u)} \leq \gamma' k^u$. Otherwise, k' would be confined to a constrained solution,

$$k' = \frac{(1-\delta)k + f(k,L) - wL - b}{1-\gamma'}. \tag{29}$$

Let k^c be the constrained solution of k'. From (4), the net profit of firms $z \geq 0$ if k^u is feasible; $z = 0$ if $k' = k^c$. The optimal k' can thereby be defined as follows,

Lemma 3.1 *In the dynamic equilibrium, the optimal* $\left(k'\right)^* = \min\{k^u, k^c\}$.

Together with the incentive compatible condition (18), the equilibrium LTV ratio γ' achieves the optimal $1-\delta$ in an unconstrained equilibrium; $\gamma' < 1-\delta$ in the constrained equilibrium.

3.2 Funding and Market Illiquidity

In constrained equilibrium, funding illiquidity can be measured by the inefficiency of γ'. If the borrowing constraint is not binding, z measures the gap between bond supply and demand. Then the funding illiquidity can be measured by the volume of the fund market imbalance [10]. Define

[10] This gap can be measured by the Order Imbalance (OIB) of the bond market.

the unit fund imbalance as μ_b. Then it equals to the borrowing margin if $k^{'} = k^u$, and unit excess debt demand if $k^{'} = k^c$, that

$$\mu_b = \frac{1-\beta\gamma^{'}}{1-\beta(1-\delta)} - \frac{z}{k^{'}} \tag{30}$$

Also define μ_d as the markup of DM goods to measure market illiquidity. Since one dollar is worth $\frac{A}{w}\beta\phi^{'}$ units of utility, the marginal cost in terms of dollars is $c_y \frac{w}{A}\frac{1}{\phi^{'}\beta}$, and the dollar price of DM goods y is $\frac{m^{'}}{y}$. In summary,

$$\mu_d = \frac{\frac{m^{'}}{y}}{c_y \frac{w}{A}\frac{1}{\phi^{'}\beta}} = \frac{\frac{A}{w} g\left(y,k^{'}\right)}{y c_1\left(y,k^{'}\right)} \tag{31}$$

Proposition 3.2 *If* $\frac{g_k\left(y,k^{'}\right)}{g\left(y,k^{'}\right)} < \frac{c_{yk}\left(y,k^{'}\right)}{c_y\left(y,k^{'}\right)}$*, then* $|\mu_b|$ *and* μ_d *move together in the equilibrium, which implies positive co-movement between funding and market liquidity* [11].

Comparing to previous literature, the co-movement of funding and market liquidity is positive no matter borrowing constraints are binding or not. This is because the co-movement is driven by agents' interdependent decisions across the first and second sub-period. Firm's investment decision in the first sub-period affects the production cost and the market intensity of the DM goods. On the other side, the ease of DM goods trade affects the pledge-ability of the collateral and the expected return on capital investment.

3.3 Steady State Equilibrium

In the dynamic equilibrium, $\phi^{'} = \phi d/\left(d_{-1}+\tau\right)$, where d_{-1} is last period's DM goods payment. Since it's optimal to spend all the money in the second sub-period, d represents the money demand in equilibrium. In a steady state equilibrium, real money demand $\phi M^{'}$ and the payment d are constant, hence $\frac{\phi}{\phi^{'}} = \nu = 1+\pi$. The Fisher equation implies that $\frac{\nu}{\beta} = (1+\pi)(1+r) = 1+i$. Then (26) in the stationary equilibrium is

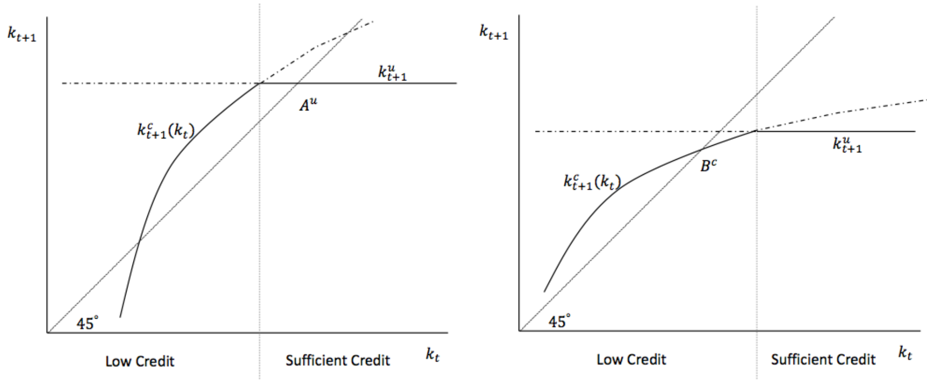
$$\frac{i}{\alpha_h} = \frac{w^s}{A}\frac{u^{'}(y^s)}{g_1(y^s,k^s)} - 1 \tag{32}$$

where the superscript s indicates steady state solution.

Figure 1 illustrates the dynamic of $\{k_t\}$. The left panel shows the scenario that only the unconstrained equilibrium A^u is supported as a steady state equilibrium; and the right panel shows the scenario that only the constrained equilibrium B^c exists in steady state. The latter could be observed if the matching friction is very high, or the labor disutility parameter A is very large.

[11] See Appendix for the proof.

Figure 1: Dynamic Evolution Of Capital



4 Quantitative Analysis

4.1 Calibration

In this section, the model is calibrated by minimizing the distance between data and model moments in steady state. Consider CRRA utility functions for households, $U(x) = \lambda logx$ and $u(y) = \frac{y^{1-\rho}}{1-\rho}$, where $\lambda > 0$ and $0 < \rho < 1$. Similar to Aruoba et al. [2009], the production technology has functional forms as follows: the production function of CM goods is $f(k, L) = k^a L^{1-a}$ and cost function of DM goods is $c(y, k) = y^{(1-\chi)} k^{\chi}$, where $0 < a < 1$ and $\chi < 0$.

There are 9 parameters to be estimated, R, ν,δ, λ, ρ, a, χ, θ, A, where four of them can be determined independently as follows. The discount factor $\beta = \frac{1}{R}$ is pinned down by the real interest rate. Gross growth rate of money supply, ν, matches the inflation rate. Capital depreciation rate, δ, is set to match the investment and capital ratio $\frac{I}{K} \approx 0.09$ [12]. Without loss of generality, normalize $\lambda = 1$.

To estimate preference parameters ρ and A, production parameters a and χ, and bargaining power θ, the plan is to match the following targets jointly: 1) the money demand to GDP ratio M^d/Y; 2) labor's share of income LS; 3) the capital stock to GDP ratio $\frac{K}{Y}$; 4) the interest rate elasticity of investment; 5) the interest rate elasticity of money demand [13]. Use the previous steady state equilibrium results, the real GDP is $Y = \int_f [f(k, L) + \alpha_f d]$, and the real money balance is $M^d = \int_h \phi m' = n\nu d$. Then the Money-to-GDP ratio is

$$\frac{M^d}{Y} = \frac{n\nu d}{f(k, L) + \alpha_f d} \tag{33}$$

The labor share of income is $LS = \frac{wL}{f(k,L)+\alpha_f d}$, and the capital output ratio is $\frac{K}{Y} = \frac{k}{f(k,L)+\alpha_f d}$.

[12] The data moments, investment-to-capital ratio, labor share of income, interest rate elasticity of investment and money demand, are taken from Aruoba et al. [2009].

[13] The money demand-to-GDP ratio, capital-to-GDP ratio, real interest rate and inflation are calculated using FRED data 1980-2011.

The two partial derivatives, $\frac{\partial k'}{\partial i}$ and $\frac{\partial y}{\partial i}$, can be calculated from the equilibrium conditions.

$$\begin{bmatrix} \frac{\partial k'}{\partial i} \\ \frac{\partial y}{\partial i} \end{bmatrix} = \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where $b_{11} = \alpha_h \frac{w'}{A} \frac{\partial\left(\frac{u_y}{g_y}\right)}{\partial k}$, $b_{12} = \alpha_h \frac{w'}{A} \frac{\partial\left(\frac{u_y}{g_y}\right)}{\partial y}$. In unconstrained equilibrium, $b_{21} = \frac{1}{R} f_{kk} - \alpha_f c_{kk}$ and $b_{22} = -\alpha_f c_{yk}$; while in constrained equilibrium $b_{22} = 0$ and $b_{21} = f_k - \delta - \frac{\partial x}{\partial k}$. Then the elasticity of investment with respect to interest rate is, $\eta_k = \frac{\partial \delta k'}{\partial i} \frac{i}{\delta k'} = i \frac{\partial k'}{\partial i}$. And the interest rate elasticity of money is $\eta_m = \frac{\partial \phi' m'}{\partial i} \frac{i}{\phi' m'}$. Since $\beta \phi' m' = g\left(y, k'\right)$, $\eta_m = \left(g_y \frac{\partial y}{\partial i} + g_k \frac{\partial k'}{\partial i}\right) \frac{i}{g}$,

$$\eta_m = \left(g_y \frac{\partial y}{\partial i} + g_k \frac{\partial k'}{\partial i}\right) \frac{i}{g} = \frac{g_k b_{22} - g_y b_{21}}{b_{11}b_{22} - b_{12}b_{21}} \frac{i}{g} \tag{34}$$

Depending on parameter values, there are 4 equilibrium regimes, which are featured by market intensity n and optimal capital k': $\{n = 1, k' = k^u\}$, $\{n = 1, k' = k^c\}$, $\{n < 1, k' = k^u\}$, $\{n < 1, k' = k^c\}$. Given the data moments specified earlier, an unconstrained equilibrium with $n = 1$ can be identified. Column BS of Table 1 shows calibrated parameters, and column BS of Table 2 summarizes model fit in terms of the target moments.

4.2 Temporary Monetary Shock Experiment

Now we introduce a one time monetary shock to the steady state equilibrium and study how liquidity responds to it. Using the above calibrated parameters, a 10% monetary shock would decrease $|\mu_b|$ by 0.0008% and μ_d by 0.14%. The market liquidity increases slightly as a response, but only temporarily. The magnitude and persistency of the impulse responses vary with parameter values. If households are less risk averse, a one-time monetary shock would be more effective in stimulating liquidity. The relative risk aversion of households in the second sub-period is $\rho = -\frac{u''(y)y}{u'(y)}$. If ρ is smaller, households are more risk averse. Consider $\rho = 0.3$ instead of the calibrated value 0.751. The steady state equilibrium still falls into the same regime. If the economy is hit by a 10% temporary monetary shock, the liquidity would immediately increase by 8%, then gradually converge back to the original steady state as shown in Figure 2. By varying χ, we can observe how the marginal productivity affects the response of liquidity to monetary shocks. The calibrated $\chi = -0.250$ has small and hence low marginal productivity of capital. A 10% monetary shock only lead to a trivial change of the liquidity. If we increase the absolute value of χ significantly, a small shock would change the optimal capital investment violently. Then the economy may oscillate between an unconstrained and constrained equilibrium. The bargaining power of households θ does not change the response of the liquidity to monetary shocks if $n = 1$. If $n < 1$, different value of θ matters if the participation n changes accordingly. Besides, the calibration is sensitive to the value of the bargaining power θ. Hence the calibrated θ is not robust with changes of interest rate.

Figure 2: IR of illiquidity to 10% one-time monetary shock

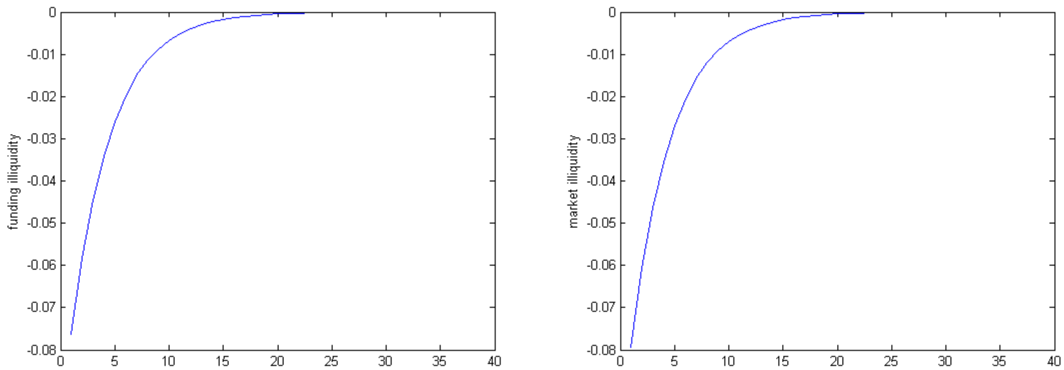
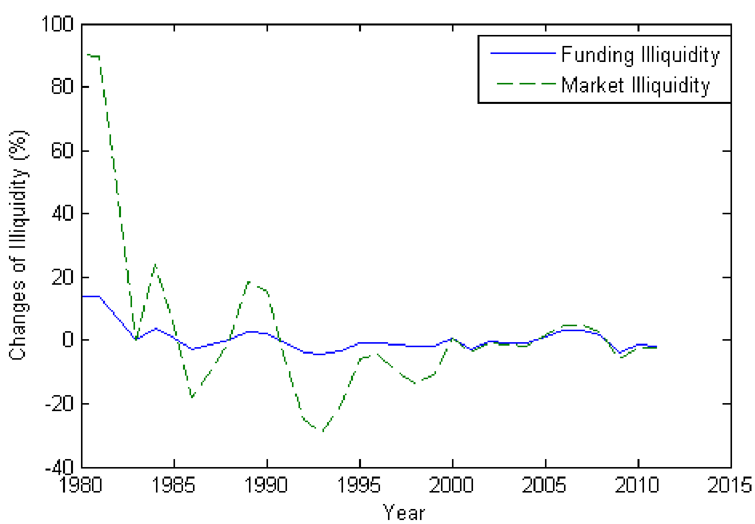


Table B and Table B show the calibration with three exogenous values of θ, 0.1,0.5 and 1. The model fit performs better if $\theta = 0.5$ and $i = i^{low} = 1.05$.

4.3 Co-movement

Consider a high interest rate steady state in the period of 1980 to1999, $i^{high} = 1.10$; and a low interest rate steady state in from 2000 to 2011,$i = i^{low} = 1.05$. To approximate how monetary shocks drive funding and market liquidity together, I use log-linearization to approximate the percentage deviation around the steady state. Figure 3 shows the simulated changes of funding and market illiquidity. In Figure 3, funding and market liquidity move together across time and present business cycle property. For example, the illiquidity jumps up during the saving and loans crises in the early 1980s and1990s. The illiquidity also increases during the period of the early 2000s recession and the recent financial crisis. Besides, the simulated funding liquidity is less volatile than the market liquidity, which is consistent with the stylized fact.

Figure 3: Percentage Changes of Funding and Market Illiquidity 1980 -2011



5 Monetary Policy

5.1 Injecting Money to Households

The conventional monetary policy tends to implement short-run goals by affecting the interest rate. In the Kiyotaki-Wright style money search model, this type of policy can be well captured by the lump-sum money injection τ to households, which targets a certain interest rate i without directly intervene financial markets. If i increases, money would depreciate and become more costly for households to carry to the second sub-period. This would lead to a real balance effect that households decrease their money holding to reduce the expected hold-up cost. On the other hand, if the real money holding decreases, money would become scarce and hence more likely to be traded. This is known as the trading opportunity effect that encourage households to carry more money and discourage the investment $k^{'}$ because of the trading opportunity effect. In summary, inflation hurts the intensive margin of trade, y and d, because of the real balance effect, but may increase the extensive margin, n, if n is not binding.

Proposition 5.1 *If $\frac{u^{'}(y)}{g_y\left(y,k^{'}\right)}$ is strictly decreasing in y and $k^{'}$, then $\frac{\partial k^{'}}{\partial i} < 0$ and $\frac{\partial y}{\partial i} \leq 0$ in steady state equilibrium. $\frac{\partial y}{\partial i} = 0$ in constrained steady state equilibrium.*

Assumption 3.1 assumes $\frac{u^{'}(y)}{g_y\left(y,k^{'}\right)}$ strictly decreasing in y. The assumption in Proposition 5.1 can be easily satisfied with the convex technology with concave $f(k,L)$ and convex $c\left(y,k^{'}\right)$. If $g_{yk} > 0$, $\frac{u^{'}(y)}{g_y\left(y,k^{'}\right)}$ would be decreasing in $k^{'}$.

Proposition 5.2 *The Friedman rule is the optimal monetary policy that maximizing y and $k^{'}$ in steady state equilibrium. But socially efficient allocations can not be achieved, that $k^{'} < \tilde{k}$ and $y < \tilde{y}$.*

Proposition 5.2 is a straightforward application of Proposition 5.1. Since the optimal $k^{'}$ and y are decreasing in i, the optimal monetary policy should set $i = 0$. To achieve socially optimum, $\tilde{y}$, $\tilde{k}$ and w should satisfy $u^{'}(\tilde{y}) = c_1(\tilde{y}, \tilde{k})$, $k^{'} = k^u$ and $w = A$ simultaneous. It can be shown that firms' borrowing constraints are always binding if $w = A$, $\theta = 1$ and $i = 0$, and $u^{'}(\tilde{y}) = c_1(\tilde{y}, \tilde{k})$ can not be satisfied if $\theta < 1$. Hence $k^{'} < \tilde{k}$ and $y < \tilde{y}$. The inefficiency of the decentralized equilibrium is caused by market frictions, such as quid-pro-quo of the bargaining process, and borrowing constraint of the fund market [14]. The efficiency loss of $k^{'}$ is increasing in fund and trade market friction, and amplified by the co-movement feature of the liquidity [15].

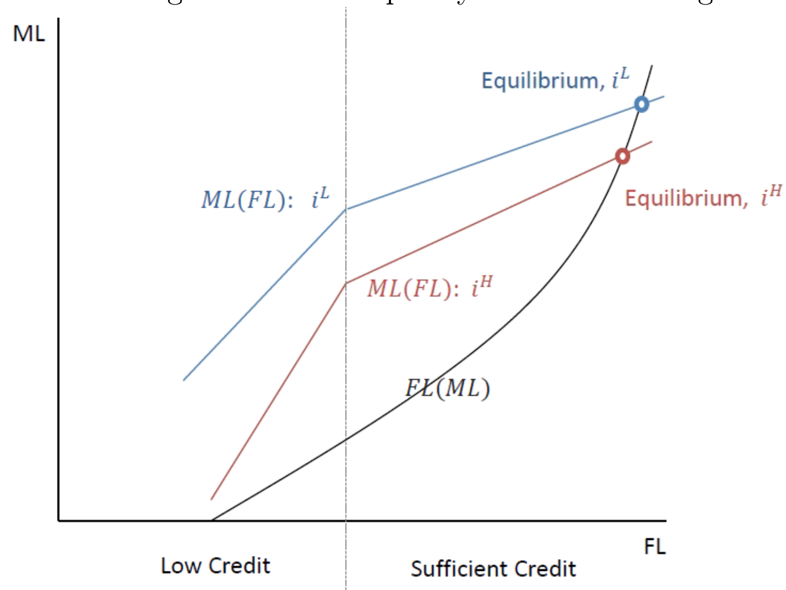
Intuitively, the response of liquidity to monetary shocks depends on the ratio between the aggregate money demand $M^{'}$ and capital $K^{'}$. A larger ratio of $M^{'}/K^{'}$ implies a bigger DM

[14] The inefficiency caused by the bargaining process would be at its minimum if $\theta = 1$.

[15] The efficiency loss of $k^{'}$ can be decomposed to two parts: the efficiency loss $k^u - k^c$ would increase if the borrowing constraint becomes tighter; the gap between the optimal $k^{'}$ in the decentralized economy and the socially optimal $\tilde{k}$ from (10), $\tilde{k} - k^u$, is increasing in the trading frictions of the second sub-period.

goods markup μ_d since more money is going after less capital proceed, and *vice versa*. If the real balance effect dominants and M'/K' decreases, then the market liquidity increases. This may happen if households have very small bargaining power and the nominal interest rate is low. In such an environment, a mild inflation may stimulate market liquidity. If the trading opportunity dominants dominants and M'/K' increases, then μ_d increases and market liquidity decreases. Figure 4 illustrate the second scenario. The co-movement between funding and market liquidity

Figure 4: Funding and market liquidity with low and high interest i



on equilibrium path is stronger if the interest rate i is higher. This result is consistent with the empirical observation that monetary policy plays a role in the co-movement between funding and market liquidity.

5.2 Injecting Money to Firms

Targeting the same interest rate i, the effects of money injection τ are the same in the long-run regardless of the injection channel. But the dynamic equilibrium path leading to the stationary equilibrium might be quite different if the injection channels are different. Consider a lump sum money injection to firms instead of households. In the short-run, the injection relaxes firms' borrowing constraint and increase the capital investment. Let Γ be the money injection to firms. Then firm's budget constraint becomes

$$z + I = f(k, L) - wL + \frac{b'}{R} - b + \Gamma. \tag{35}$$

Injection Γ can be financed by a lump sum tax from household or new money. Consider the latter one that the monetary authority increases money supply to support the liquidity injection. Assume Γ targets the same money growth rate, $\Gamma = M_{-1}(\nu - 1)$. The budget constraint of the household is (2) with $\tau = 0$. Since ϕ' remains the same as before and $\tau = 0$, injection Γ does not change the marginal decision on m' and b'. In the constrained equilibrium, firms' investment is constrained by $\gamma' k'$ where the LTV ratio γ' is inefficient. With injection Γ, not only the investment on k' increases; the LTV ratio γ' would also increase as the value of collateral increased, which further improves k'. In an unconstrained equilibrium, firms invest at optimal level k^u. Injection $\Gamma > 0$ would increase the imbalance of the fund market and hence decrease the funding liquidity.

Proposition 5.3 *On the equilibrium path of the transition,* $\frac{\partial |\mu_b|}{\partial \Gamma} \leq 0$ *and* $\frac{\partial k'}{\partial \Gamma} \geq 0$ *if capital investment is constrained;* $\frac{\partial |\mu_b|}{\partial \Gamma} \geq 0$ *and* $\frac{\partial k'}{\partial \Gamma} = 0$ *if capital investment is unconstrained.*

A special case that has not been discussed is the change of the equilibrium regime. If the injection Γ pushes the economy to change from a constrained equilibrium to an unconstrained equilibrium, the effect of the monetary policy depends on how tight the borrowing constraint is in the old equilibrium and the overshooting of the injection that increases the funding market imbalance.

5.3 Injecting Money to Financial Intermediaries

Now consider a monetary injection F to financial intermediaries. Introduce the government bond b^g as an instrument to convene the injection. Assume the government bond is naturally secured that borrowers can not default on b^g. This assumption is similar to Gertler and Kiyotaki [2010] that the government equity can not be diverted. Then firm's budget constraint becomes

$$z + I = f(k, L) - wL + \frac{b'}{R} - b + \frac{b^{g'}}{R} - b^g \tag{36}$$

Here government and private bond have the same rate of return R. If firms are constrained without government bond, (36) implies that the injection can relax the constraint and increase k'. Then more collaterals become available to secure the private loan. Financial intermediaries still require $b' \leq \gamma' k'$ and choose γ' to avoid default, that

$$\underbrace{1 - \gamma'\beta}_{Marginal\ Default\ Cost} = \underbrace{-\alpha_f c_k \left(y, \hat{k}'\right) + \frac{1}{R} f_k \left(\hat{k}', L'\right) - \frac{b^{g'}}{k'}}_{Marginal\ Default\ Gain}. \tag{37}$$

Notice that the marginal default gain decreases by $\frac{b^{g'}}{k'}$. The marginal default incentive with regard to capital decreases. Hence the LTV ratio γ' increases and funding liquidity would be improved.

To target the same money growth rate, let $F = M_{-1}(\nu - 1)$ and $\tau = 0$. The bond market clears if $\int_{\mathrm{h}} b' = \int_{\mathrm{f}} b'$ and $F \geq \int_{\mathrm{f}} b^{g'}$. If the economy is in an unconstrained equilibrium before the injection or excessive F is introduced to a constrained economy, firms' demand of government bond would be strictly less than F. Then a fraction of the injection would be held by financial intermediaries as reserve, or by firms as cash reserve. If firms hold the reserve and z increases, the increased fund imbalance would lessen the funding liquidity, which is not a desirable outcome.

6 Price Posting

Price posting is an efficient process which is consistent with Hosio's condition. Terms of trade, contracts, can be posted by firms, households or market makers. Since price posting with random search leads to equilibria/equilibrium similar to competitive equilibria/equilibrium, it's computational more appealing. Assume market makers design and post a number of contracts (y, d) before meetings in the second sub-period. Agents are able to direct their searches towards favorable terms of trade. The market tightness of a submarket with contract (y, d) is $n = \frac{h}{f}(y, d)$, where h is the measure of households applying the contract and f is the measure of firms posting the contract. Let the exogenous supply of households and firms be equally distributed. Define the matching function as $\mathcal{M}(h, f)$, then the matching probabilities for firms and households are $\alpha_f = \frac{\mathcal{M}(h,f)}{f} = \mathcal{M}(n, 1)$ and $\alpha_h = \frac{\mathcal{M}(h,f)}{h} = \frac{\mathcal{M}(n,1)}{n}$ respectively. Use the previous results for households' and firms' optimization problems. Households' problem can be simplified as

$$W^H(m, b) = U_1 + \underset{m'}{max} \left[\left(\beta \frac{A\phi'}{w'} - \frac{A\phi}{w} \right) m' + \alpha_h \left[u(y) - \beta \frac{A}{w'} d \right] \right] \tag{38}$$

where $U_1 = \frac{A\phi}{w}(m + \tau) + \frac{A}{w}b + U(x^*) - \frac{A}{w}x^* + \beta W^H(0, 0)$ [16]. Similarly, firms' problem can be rearranged as

$$W(b, k) = \Pi_1 + \underset{b',k'}{max} \left\{ -k' + \beta (1 - \delta) k' + \beta f \left(k', L \right) + \alpha_f(n) \left[-c(y, k') + \beta d \right] \right\} \tag{39}$$

where $\Pi_1 = -b + (1 - \delta)k + f(k, L^*) - AL^* + \frac{1}{R}W(0, 0)$. Since x^* and L^* are independently determined, U_1 and Π_1 are constant to the optimization problems.

Market makers maximize firms' continuation value $W(b, k)$ while guaranteeing households reservation utility $\bar{U}$,

$$\begin{aligned} \underset{\{n,y,d\}}{max} \quad & W(b, k) \\ s.t. \quad & W^H(m, b^H) = \bar{U} \end{aligned} \tag{40}$$

Plug (39) and (38) into (40), the market makers' problem becomes

$$\underset{\{n,y,d,k'\}}{max} \left\{ [\beta (1 - \delta) - 1] k' + \beta f \left(k', L \right) + \alpha_f \left[-c(y, k') + \beta d \right] \right\} \tag{41}$$

$$s.t. \underset{m'}{max} \left\{ \left(\beta \frac{A\phi'}{w'} - \frac{A\phi}{w} \right) m' + \alpha_h \left[u(y) - \beta \frac{A}{w'} d \right] \right\} = \bar{U} - U_1 \tag{42}$$

Define $\bar{U} - U_1 \triangleq \hat{U}$, since $\phi' m' = d$ in equilibrium, households' individual rationality constraint

[16] Here x^* is the optimal consumption independently determined by the first order condition

(42) can be written as $\left(\beta\frac{A}{w'}-\frac{A\phi}{w\phi'}-\alpha_h\frac{A}{w'}\beta\right)d+\alpha_h u\left(y\right)=\hat{U}$. Plug it back to (42) to get

$$\underset{\{n,y,k'\}}{max}\left\{\left[\beta\left(1-\delta\right)-1\right]k'+\beta f\left(k',L\right)-\alpha_f c(y,k')+\alpha_f\frac{\hat{U}-\alpha_h u(y)}{\frac{A}{w'}-\frac{A\phi}{w\phi'\beta}-\alpha_h\frac{A}{w'}}\right\}$$

Let η_h and η_f be the matching elasticities of households' and firms' contribution. Then $\eta_h=\frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta H/H}=\frac{\alpha_f'(n)}{\alpha_h}$, $\eta_f=\frac{\Delta\mathcal{M}/\mathcal{M}}{\Delta F/F}=-\frac{n\alpha_h'(n)}{\alpha_h}$ and $\eta_f+\eta_h=1$. Using the first order conditions with respect to n and y, the optimality condition of y is

$$\beta d=\frac{\eta_h c(y,k')u'(y)+\eta_f c_1(y,k')u(y)}{\eta_h u'(y)+\eta_f\frac{A}{w'}c_1(y,k')}\triangleq\tilde{g}(y,k') \tag{43}$$

Definition 6.1 *In the Competitive Search Equilibrium, $\{k',n,y,d\}$ solves the market makers problem (40). The DM goods market clears such that $\int n(y,d)f(y,d)=1$.*

The market tightness $n(\hat{U})$ is strictly decreasing in $\hat{U}$. If $\hat{U}=0$, the household is indifferent between participating the second sub-period or not. In the equilibrium, every sub-market behaves the same in which firms always participate. If $n(0)>1$, market tightness is constrained that $n(\hat{U})=1$, $\hat{U}=n^{-1}(1)$ and $\bar{U}=\hat{U}+U_1$. If $n(0)\leq 1$, then $\hat{U}=0$ and $\bar{U}=U_1$. The existence can be guaranteed by the following participation constraints of firms.

Claim 6.1 *A monetary equilibrium exists if and only if the optimal y and k' are greater than zero if $n=n(0)$, that*

$$\underset{\{n,y,k'\}}{max}\left\{\left[\beta\left(1-\delta\right)-1\right]k'+\beta f\left(k',L\right)+\alpha_f\left[-c\left(y,k'\right)+\frac{\hat{U}-\alpha_h u(y)}{\frac{A}{w'}-\frac{A\phi}{w\phi'\beta}-\alpha_h\frac{A}{w'}}\right]\right\}>0.$$

In steady state, k', y and n are decided by Lemma 3.1 and the following conditions

$$i=\alpha_h(n)\left(\frac{w}{A}\frac{u'(y)}{c_1(y,k')}-1\right) \tag{44}$$

$$\frac{\alpha_h(n)u(y)-\hat{U}}{i+\alpha_h(n)}=\tilde{g}(y,k') \tag{45}$$

If $i=0$, (45) implies $\frac{w}{A}u'(y^0)=c_1(y^0,k)$. Similar to the bargaining solution, Friedman rule is the optimal monetary policy. The optimality conditions are shown in the appendix. Here the previous exogenous bargaining power θ is replaced by the endogenous matching elasticity η_h. Since capital k' has unique interior solution, the calibrated parameters of this model are more robust than the bargaining model. Columns 'PP' of Table 1 to B show the calibrated parameters and moments using price posting equilibrium. On average, price posting matches data better than bargaining model. Although price posting model tends to overestimate the money demand, especially in low interest rate environment.

7 Conclusion

In the liquidity literature, the time-series properties of the co-movement between funding and market liquidity and common factors that drive this co-movement have remained largely unexplored. One explanation suggests that borrowers would buy more collateralized assets if the borrowing constraints are relaxed since they are always borrowing constrained in equilibrium. The increase of asset supply would decrease the markup of the asset, and hence increase the market liquidity. This paper introduces money and capital to connect the fund market and DM goods market in both constrained and unconstrained equilibrium. Capital $k^{'}$ is both collateral for secured loans and input factor for goods production. If the trading market becomes more liquid, firms would be more profitable. More pledge-able profit increases the pledge-ability and hence the funding liquidity. If the fund market is more liquid that firms could build up a larger capital stock, the total gain from trade would increase as the marginal production cost decreases. Meanwhile, holding capital is costly as long as the depreciation rate is strictly positive. Firms would lower price markups to increase the probability of trade and reduce the hold-up cost of capital. Hence market liquidity increases.

Calibrated the model, we can quantitatively study the liquidity co-movement and experiment monetary shocks using first order disturbance. The impulse responses of liquidity to one-time monetary shocks are quite different as parameters values varying. For example, with small risk averse coefficient ρ or large productivity parameter $|\chi|$, the one-time shock can increase liquidity for a certain period of time before it converges back to the steady state.

If the monetary authority injects money to the economy to target a higher interest rate, the money demand may increase or decrease, depending on the real balance and trading opportunity effect. If the money to capital ratio increases, liquidity would decrease; and *vice versa*. Therefore liquidity respond to monetary shocks even if firms' optimal capital is not constrained and not affected by monetary shocks directly. In the long-run, higher interest rate decreases capital $k^{'}$ and output y, which hurts the intensive margin. On the other side, mild inflation may increase the extensive margin n, in that the trading opportunity increases. This can be observed with very small θ and $n < 1$. In the short run, the transition paths to the new equilibrium depends on whether money is injected to households, firms or financial intermediaries. Injecting more money to firms instead of households increases the capital and funding liquidity if the borrowing constraint is binding. If not binding, the injection would decrease funding liquidity since it aggravate the fund imbalance. If the monetary authority injects money through financial intermediaries, the fund and trading market would be more liquid in the short-run, but the reserve may increase dramatically if the borrowing constraint is binding.

In the extended model, the price posting mechanism internalizes the bargaining power θ. The market tightness n depends on households' reservation utility $\hat{U}$. Given $\hat{U}$, market tightness n, capital $k^{'}$, and DM goods production y are decided simultaneously, which yield a more robust result numerically. Although this mechanism avoids the inefficiency of the bargaining process, the major results of liquidity co-movement and efficiency loss still hold.

In summary, the responses of liquidity to monetary shocks rely heavily on parameter values of preference, production, bargaining power and *etc.* Without an accurate justification of the environment and the underline mechanism, monetary shocks may bring unintended consequences such as inefficient production or higher trading frictions.

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A Proof

A.1 Properties of $g\left(y, k^{'}\right)$

$g\left(y, k^{'}\right)$ solves the Nash Bargaining problem in section 2.6, that

$$g\left(y, k^{'}\right) = \frac{\theta u_y c(y, k^{'}) + (1-\theta) c_y u(y)}{\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y}$$

If the total surplus of trade is positive, $u(y) - \frac{A}{w^{'}} c(y, k^{'}) > 0$, then $c\left(y, k^{'}\right) < g\left(y, k^{'}\right) < \frac{w^{'}}{A} u\left(y\right)$.

$$g = c + \frac{(1-\theta)\left(c_y u - \frac{A}{w^{'}} c_y c\right)}{\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y} = \frac{w^{'}}{A} u - \frac{\theta\left(\frac{w^{'}}{A} u_y u - u_y c\right)}{\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y}$$

Take first order derivative with regard to y and $k^{'}$

$$g_y\left(y, k^{'}\right) = \frac{\theta u_y c_y}{\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y} + \frac{\theta\left(1-\theta\right)\left[u(y) - \frac{A}{w^{'}} c(y, k^{'})\right]\left(c_{yy} u_y - c_y u_{yy}\right)}{\left[\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y\right]^2} \tag{46}$$

$$g_k\left(y, k^{'}\right) = \frac{\theta u_y c_k}{\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y} + \frac{\theta\left(1-\theta\right)\left[u(y) - \frac{A}{w^{'}} c(y, k^{'})\right] u_y c_{yk}}{\left[\theta u_y + (1-\theta)\frac{A}{w^{'}} c_y\right]^2} \tag{47}$$

Concavity assumptions of the utility and technology implies that $u_y > 0$, $u_{yy} < 0$, $c_y > 0$, $c_{yy} > 0$, $c_k < 0$, $c_{kk} > 0$, $c_{yk} < 0$. Apply to equations (47) and (47), there are $g_y\left(y, k^{'}\right) > 0$ and $g_k\left(y, k^{'}\right) < 0$. The total derivatives $dg\left(y, k^{'}\right) = g_y dy + g_k dk^{'}$ implies that $\frac{\partial k^{'}}{\partial y} > 0$ if $dg\left(y, k^{'}\right) = 0$.

Besides, if y is less than efficient level of output y^{*}, then $\frac{w^{'}}{A} u_y > c_y$ and the first term of (47) is greater then c_y. Since the second term of (47) is positive, $g_y > c_y$. Similarly we can get $g_k < c_k < 0$ if $y < y^{*}$.

A.2 Proof of Proposition 3.1. Existence of Monetary Equilibrium

Proof. In a monetary equilibrium, the money demand is greater than zero. In this model, it means that the households' money holding at the beginning of the second sub-period, $m^{'} = d/\phi^{'}$ is strictly positive. In the following proof, I will show the existence of the equilibrium first, and then show the equilibrium money demand $d/\phi^{'} = g\left(y, k^{'}\right)/\beta\phi^{'}$ is positive in the equilibrium. Both households and firms understand the Nash Bargaining Process that deciding y and $d = g\left(y, k^{'}\right)/\beta$. Households take $k^{'}$ as given and maximize the expected gain of the second sub-period trade as follows.

$$G\left(y; k^{'}\right) \triangleq \underset{y}{max}\left\{-\frac{\phi^{'}}{\phi} g(y, k^{'}) + \alpha_h\left[u(y) - \frac{A}{w} g(y, k^{'})\right]\right\} \tag{48}$$

where other variables, such as $k^{'}$, w and L optimize first sub-period problem taking y as given. The first derivative of (48) is

$$G_y = -\frac{\phi^{'}}{\phi} g_y(y, k^{'}) + \alpha_h \left[u_y - \frac{A}{w^{'}} g_y(y, k^{'}) \right] \tag{49}$$

Divide both sides by u_y and rearrange it, then

$$\frac{G_y}{u_y} = \alpha_h - \left(\frac{\phi^{'}}{\phi} + \alpha_h \frac{A}{w^{'}} \right) \frac{g_y(y, k^{'})}{u_y} \tag{50}$$

Assumption 3.1 assumes that $\frac{g_y}{u_y}$ is strictly increasing in y, hence the right hand side of (50) is strictly decreasing in y. On the right hand side, since u_y is increasing in y, G_y must be decreasing in y that $G_{yy} < 0$.

Given optimal y, the first sub-period optimization can be summarized as the following optimization problem

$$F\left(\mu, \lambda, k^{'}, L; y\right) \triangleq \max_{\mu, \lambda, k^{'}, L} z + \mu z + \lambda \zeta + \alpha_f \left[g\left(y, k^{'}\right) - c\left(y, k^{'}\right) \right] \tag{51}$$

$$s.t. \quad \zeta = \alpha_f g\left(y, k^{'}\right) / \beta + \left(U^{'}\right)^{-1} \left(\frac{A}{w}\right) - (1-\beta) \gamma^{'} k^{'} - wL \tag{52}$$

$$z = f(k, L) + \frac{\gamma^{'} k^{'}}{R} - wL - \left[k^{'} - (1-\delta) k \right] \tag{53}$$

where multipliers $\lambda \geq 0$, $\mu \geq 0$, and $\lambda z = 0$ in equilibrium [17]. Applying the convex technology and concave utility function assumption, it's straight forward that $F\left(\mu, \lambda, k^{'}, L; y\right)$ is also continuous and concave. According to Nash's existence Theorem, the continuity and concavity of (48) and (51) guarantees the existence of the Nash Bargaining Equilibrium.

Moreover, $g\left(y = 0, k^{'}\right) = 0$ for any $k^{'}$, hence $G(y = 0) = 0$ if $u\left(y = 0\right) = 0$. If (49) is non-positive, then y would have a corner solution zero. It would violate the assumption that (48) is strictly positive. Hence (49) is positive, and $y > 0$ in equilibrium. Hence $g\left(y > 0, k^{'}\right) > g\left(0, k^{'}\right) = 0$, and monetary equilibrium exists. ■

[17] Other controls are determined as follows: $U^{'}(x) = \frac{A}{w}$,$w^{'} = w$, $w = f_L(k, L)$, $b^{'} = \gamma^{'} k^{'}$, $d = g(y, k^{'})/\beta$, $m^{'} = \frac{d}{\phi^{'}}$, $I = k^{'} - (1-\delta)k$, $\frac{1}{\phi^{'}} = \frac{\phi(m+\tau) + \alpha_f d}{\phi d}$.

A.3 Proof of Proposition 3.2, co-movement of funding and market liquidity

If $\frac{g_k}{g} < \frac{c_{yk}}{c_y}$, the markup of one dollar in the second sub-period $1+\mu_d = \frac{m'}{y}/c_y \frac{w}{A}\frac{1}{\phi'\beta} = \frac{A}{w} g/yc_y$ would be decreasing in k',

$$\frac{\partial(\mu_d)}{\partial k'} = \frac{(g_k c_y - g c_{yk})}{y c_y^2}\frac{A}{w'} < 0. \tag{54}$$

Let $k' = G(y, w', \mu_d)$, the funding illiquidity can be written as

$$1+\mu_b = \frac{1-\beta\gamma'}{1-\beta(1-\delta)} - \frac{z}{k'} = \frac{\frac{1}{R} f_k\left(G(y,w',\mu_d), L\right) - \alpha_f c_k\left(y, G(y,w',\mu_d)\right)}{1-\beta(1-\delta)} - \frac{z}{G(y,w',\mu_d)} \tag{55}$$

Then $\frac{\partial|\mu_b|}{\partial|\mu_d|} = \frac{\partial|\mu_b|}{\partial G}\frac{\partial G}{\partial|\mu_d|}$. Since $\mu_d > 0$, (54) implies that $\frac{\partial G}{\partial(\mu_d)} < 0$.

In the unconstrained equilibrium, $\mu_b > 0$ and $\frac{\partial(\mu_b)}{\partial G} = \frac{1}{1-\beta(1-\delta)}\left[\frac{1}{R} f_{kk} - \alpha_f c_{kk}\right] < 0$. In the constrained equilibrium, $\mu_b = -z/G < 0$, $\frac{\partial(\mu_b)}{\partial G} = \frac{\partial}{\partial G}\left(\frac{z}{G}\right) > 0$. Hence $\frac{\partial|\mu_b|}{\partial G} < 0$ and $\frac{\partial|\mu_b|}{\partial|\mu_d|} > 0$.

Besides, we want to show that funding illiquidity is increasing in the market illiquidity of last period, $\frac{\partial|\mu_b|_t}{\partial|\mu_d|_{t-1}} > 0$. From the previous results, $\frac{\partial G\left(y_{t-1}, w, \mu_{d,t-1}\right)}{\partial|\mu_d|_{t-1}} < 0$, and $\frac{\partial|\mu_b|_t}{\partial G\left(y_{t-1}, w, \mu_{d,t-1}\right)} = \frac{\partial|\mu_b|_t}{\partial k} < 0$. In summary, funding and market liquidity moves together.

$$\begin{bmatrix} \mu_{bt} \\ \mu_{dt} \end{bmatrix} = \begin{bmatrix} \sigma_b^2 & cov(\mu_b, \mu_d) \\ cov(\mu_b, \mu_d) & \sigma_d^2 \end{bmatrix} \begin{bmatrix} \mu_{b,t-1} \\ \mu_{d,t-1} \end{bmatrix} \tag{56}$$

A.4 Proof of Proposition 5.1

If $\frac{u'(y)}{g_y(y,k')}$ is strictly decreasing in y and k', then $\frac{\partial k'}{\partial i} < 0$ and $\frac{\partial y}{\partial i} \le 0$ in steady state equilibrium. $\frac{\partial y}{\partial i} = 0$ in constrained equilibrium.

Proof. The optimality conditions imply that $\frac{\partial y}{\partial i}$ and $\frac{\partial k'}{\partial i}$ are determined

$$B\begin{bmatrix} \frac{\partial k'}{\partial i} \\ \frac{\partial y}{\partial i} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where

$$B = \begin{bmatrix} \alpha_h \frac{w'}{A}\frac{\partial\left(\frac{u_y}{g_y}\right)}{\partial k} & \alpha_h \frac{w'}{A}\frac{\partial\left(\frac{u_y}{g_y}\right)}{\partial y} \\ \frac{1}{R} f_{kk} - \alpha_f c_{kk} & -\alpha_f c_{yk} \end{bmatrix}$$

in unconstrained equilibrium.

$$\begin{bmatrix} \frac{\partial k^{'}}{\partial i} \\ \frac{\partial y}{\partial i} \end{bmatrix} = B^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{b_{11}b_{22} - b_{12}b_{21}} \begin{bmatrix} b_{22} & -b_{12} \\ -b_{21} & b_{11} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Using the assumptions and properties showed earlier, we can get $b_{11} < 0$, $b_{12} < 0$, $b_{21} < 0$, and $b_{22} > 0$. Hence $\frac{\partial k^{'}}{\partial i} < 0$ and $\frac{\partial y}{\partial i} < 0$.

In the constrained equilibrium, $b_{22} = 0$ and $b_{21} = f_k - \delta - \frac{\partial x}{\partial k} > 0$ since $f_k > \delta$ and $\frac{\partial x}{\partial k} = \frac{1}{U^{'}(x)} \left(-\frac{A}{f_L^2} f_{Lk} \right) < 0$. Hence $\mid B \mid > 0$, $\frac{\partial k^{'}}{\partial i} = 0$ and $\frac{\partial y}{\partial i} < 0$. ∎

A.5 Proof of Proposition 5.3.

In the constrained competitive equilibrium, $\frac{\partial |\mu_b|}{\partial \Gamma} \leq 0$, $\frac{\partial k^{'}}{\partial \Gamma} \geq 0$; in the unconstrained equilibrium, $\frac{\partial |\mu_b|}{\partial \Gamma} \geq 0$ and $\frac{\partial k^{'}}{\partial \Gamma} = 0$.

Proof. In constrained equilibrium,

$$k^c = \frac{(1-\delta)\, k + f\,(k, L) - wL - b + \Gamma}{1 - \beta\gamma^{'}} \tag{57}$$

$$1 - \beta\gamma^{'} = -\alpha_f c_k\,(y, k^c) + \beta f_k\,(k^c, L) \tag{58}$$

Plug (58) into (57) and take derivative with respect to Γ

$$\frac{\partial k^c}{\partial \Gamma} = \left[\frac{1}{R} f_{kk} \left(k^{'}, L^{'} \right) k^c + \frac{1}{R} f_k \left(k^{'}, L^{'} \right) - \alpha_f c_{kk} \left(y, k^{'} \right) k^c - \alpha_f c_k \left(y, k^{'} \right) \right]^{-1}$$

With the assumptions of the curvature of production function, $f_{kk}\left(k^{'}, L^{'}\right) k^c + f_k\left(k^{'}, L^{'}\right) \geq f_k\left(2k^{'}, L^{'}\right) > 0$ if kf_k is increasing in k. And $c_{kk}\left(y, k^{'}\right) k^c + c_k\left(y, k^{'}\right) < 0$ if kc_k is decreasing in k. Hence $\frac{\partial k^c}{\partial \Gamma} \geq 0$. The funding illiquidity, $|\mu_b| = \left(1 - \beta\gamma^{'}\right) / \left[1 - \beta\,(1-\delta)\right]$, is decreasing in $\gamma^{'}$, and $\gamma^{'}$ is increasing in $k^{'}$. Together with the previous result, the funding illiquidity $|\mu_b|$ is decreasing in the injection Γ. Hence the bond market becomes more liquidity.

In the unconstrained equilibrium,

$$\mid \mu_b \mid = z = f\,(k, L) + (1-\delta)k - \left[1 - \beta(1-\delta)\right] k^u - wL - b + \Gamma$$

Since Γ doesn't enter the optimality conditions, optimal k^u and L are invariant with the injection Γ. Only z is increasing with Γ, hence $\mid \mu_b \mid$ is decreasing in Γ. ∎

A.6 Price Posting Equilibrium

First order conditions specify $k^{'}$, y and n

$$
\begin{aligned}
k^{'}: &\quad \left(\frac{1-\delta}{R}-1\right)+\frac{f_1\left(k^{'},L\right)}{R}-\alpha_f(n)c_2(y,k^{'}) &= 0\\
y: &\quad \alpha_f(n)c_1(y,k^{'})+\frac{1}{R}\alpha_f(n)\frac{\alpha_h(n)u^{'}(y)}{\hat{\beta}-\frac{A\phi}{w\phi^{'}}-\alpha_h(n)\hat{\beta}} &= 0\\
n: &\quad -\alpha_f^{'}(n)c(y,k^{'})+\frac{1}{R}\alpha_f^{'}(n)\frac{\hat{U}-\alpha_h(n)u(y)}{\hat{\beta}-\frac{A\phi}{w\phi^{'}}-\alpha_h(n)\hat{\beta}}+\frac{1}{R}\alpha_f(n)\frac{\alpha_h^{'}(n)\left[\hat{\beta}\hat{U}-\hat{\beta}u(y)+\frac{A\phi}{w\phi^{'}}u(y)\right]}{\left[\hat{\beta}-\frac{A\phi}{w\phi^{'}}-\alpha_h(n)\hat{\beta}\right]^2} &= 0
\end{aligned}
$$

Combine the last two

$$
\begin{aligned}
d = \phi^{'}m^{'} &= \frac{-\alpha_f^{'}(n)c(y,k^{'})u^{'}(y)+n\alpha_h^{'}(n)c_1(y,k^{'})u(y)}{-\alpha_f^{'}(n)\frac{1}{R}u^{'}(y)+n\alpha_h^{'}(n)\hat{\beta}c_1(y,k^{'})}\\
&= \frac{\eta_h c(y,k^{'})u^{'}(y)+\eta_f c_1(y,k^{'})u(y)}{\eta_h u^{'}(y)+\eta_f\frac{A}{w^{'}}c_1(y,k^{'})}
\end{aligned}
$$

B Tables

Table 1: Calibrated Parameters

Production	**BS**	**PP**	**Description**
δ	0.07	0.07	Capital Depreciation
a	0.201	0.225	CM Goods Production - Capital Intensity
χ	-0.250	-122.372	DM Goods Production - Cost Function Parameter
Preference	**BS**	**PP**	**Description**
λ	1	1	Utility Parameter of CM Goods Consumption - Scalar
ρ	0.751	0.113	Utility Parameter of CM Goods Consumption - Relative Risk Aversion
A	78.955	5.591	Disutility of Work
Policy and Other	**BS**	**PP**	**Description**
R	1.036	1.036	Real Interest Rate
ν	1.050	1.050	Growth Rate of Money Supply
θ	0.487	-	Buyer's Bargaining Power of DM Goods Trades
$\hat{U}$	-	0.186	Households' Reservation Utility

Table 2: Calibrated Moments

Target Moments	**Data**	**BS**	**PP**	**Description**
M^d/Y	0.130	0.148	0.151	Money Demand
K/Y	1.797	1.793	1.793	Capital-to-Output Ratio
LS	0.710	0.743	0.705	Labor Share of Income
η_I	-0.0023	-0.0124	-	Labor-to-Capital Ratio
η_m	-0.226	-0.355	-0.223	Interest Elasticity of Money

Table 3: Calibrated Parameters

	i^{low}				i^{high}			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Par	$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$	PP	$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$	PP
a	0.225	0.230	0.255	0.233	0.386	0.086	0.120	0.28
χ	-2.486	-0.916	-0.107	-58.002	-0.047	-0.878	-0.828	-30.230
ρ	0.681	0.674	0.999	0.126	0.989	0.429	1	0.177
A	4711.481	20.738	11.985	5.988	0.276	4.397	141.203	5.882
ν	1.0149	1.0149	1.0149	1.0149	1.0617	1.0617	1.0617	1.0617
$\hat{U}$	-	-	-	0.142	-	-	-	0.187

$R = 1.036$, $\delta = 0.9$,$\lambda = 1$

Table 4: Moments

	i^{low}					i^{high}				
Moments	Data	(1)	(2)	(3)	(4)	Data	(5)	(6)	(7)	(8)
		$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$	PP		$\theta = 0.1$	$\theta = 0.5$	$\theta = 1$	PP
M^d/Y	0.11	0.120	0.118	0.397	0.1453	0.13	0.874	0.493	0.326	0.150
K/Y	1.797	1.798	1.799	1.797	1.791	1.797	1.821	1.815	1.813	1.794
LS	0.71	0.729	0.725	0.599	0.698	0.71	0.361	0.701	0.745	0.656
η_m	-0.226	0.0005	-0.0176	-0.0226	-0.226	-0.226	0.011	0.184	-0.043	-0.216