Excess Liquidity against Predation

by

Dai Zusai

Department of Economics

DETU Working Paper 12-01

June 2012

1301 Cecil B. Moore Avenue, Philadelphia, PA 19122

http://www.cla.temple.edu/economics/index.htm

Excess Liquidity against Predation

Dai Zusai*

June 28, 2012

Abstract

We consider precautionary liquidity holding as counter-strategy for the entrant to protect himself from predation. Threat of predation, even if avoided in equilibrium, affects the financial contract to raise precautionary liquidity and the equilibrium outcome in the product market competition. When the incumbent's strategy is unverifiable, the entrant with small start-up capital cannot raise large enough precautionary liquidity; consequently, he shrinks his business so as to avoid predation. Predation evolves in the model only as perturbation from equilibrium strategy. We provide the revelation principle for a sequential equilibrium to select a sensible outcome by imposing robustness to such perturbation.

Keywords: predation, excess liquidity, revelation principle, sequential equilibrium, strategic uncertainty

JEL classification: L12; D86; G30

*Department of Economics, Temple University, 1301 Cecil B. Moore Ave., RA 873 (004-04), Philadelphia, PA 19122, U.S.A. Tel:+1-215-204-1762. E-mail: `zusai@temple.edu`. I gratefully acknowledge Noriyuki Yanagawa, Toshihiro Matsumura, and Bill Sandholm for their encouragement. I also thank Munetomo Ando, Daisuke Oyama, Ikuo Ishibashi, Susumu Cato, Raymond J. Deneckere, Marek Weretka, Dimitrios Diamantaras and the seminar participants at U.Tokyo, Hitotsubashi U., Kyoto U., U.Wisconsin-Madison, Japan Fair Trade Committee, Japan Economic Association, and Hakone Microeconomic Conference for helpful comments and discussions.

1 Introduction

It has been long argued in antitrust policy and industrial economics whether predation by a "long-purse" incumbent to a low-capitalized entrant distorts economic outcomes in equilibrium. If all the agents are rational, the entrant could raise enough precautionary liquidity to survive predation; anticipating it, the incumbent would not try predation. This is the classic "long-purse" theory raised by Telser (1966).[1] After the 1993 case *Brooke Group v. Brown & Williamson Tobacco*[2], such negative view on predation is dominant in US antitrust courts (Elzinga and Mills, 2001).

However, this reasoning leaves out the questions of how the entrant can attain sufficient liquidity to avoid predation and whether the entrant's output decision can be totally independent from the liquidity demand. While the modern financial theory of predation such as Bolton and Scharfstein (1990), based on contract theory of incomplete information games, has verified possibility of predation against financially weak entrants, it assumes fixed costs or physical uncertainty as the source of liquidity demand and thus does not give direct answer to these questions. Lerner (1995) observes in the 1980s disk drive industry that the slump in capital market triggers predatory price cut against entrants with small internal capital. This empirical study suggests a link between difficulty in financing and vulnerability to predation.

In this paper, we present endogenous link between threat of predation and demand of liquidity. Further, we show that, if realization of predation is unverifiable, the entrant with small start-up capital cannot raise enough precautionary liquidity to completely invalidate threat of predation and thus the product market outcome is distorted.

In our model, an entrant and an incumbent compete in a product market *à la* Cournot: they simultaneously set capacity sizes, which we can interpret in general as precommitted determinants of their competence in the market. While there is no fixed cost and no physical uncertainty, the only essential departure of our production market from Cournot competition is that the entrant faces a cash-in-advance constraint on the payment of capacity cost: he has to pay it before achieving the sales and otherwise he must exit the market.[3] To pay it

[1] Precisely speaking, Telser (1966) predicts that a rational incumbent rather tries to buy out the entrant, not just giving up the monopoly profit. Also he suggests that the incumbent should use threat of predation to reduce the take-over bid of the entrant's company. But, since Telser's model takes the amount of precautionary liquidity ('reserve') as exogenously given, he concludes that an entrant should have plenty of liquidity to raise the take-over bid.

[2] Brooke Group Ltd. v. Brown & Williamson Tobacco Corp., 509 U.S. 209 (1993).

[3] We do not allow the incumbent to reset the capacity size after the exit; so we would see their choice variables as precommitted determinants of competence like a capacity, rather than production level.

out, he can ask for a short-term loan by mortgaging future sales or just receiving advance draw of sales; future sales have become certain (though maybe unverifiable) since capacity sizes are set. This triggers threat of predation: the incumbent would set excess capacity size to reduce the entrant's future sales and to prevent him from obtaining a large enough additional loan to pay out the capacity costs.

Precautionary liquidity on the time of entry lets the entrant less rely on the short-term loan and survive predation. Thus it invalidates the threat of predation and the incumbent does not try predation. The entrant uses it as a device to commit himself to staying on the market. So the threat of predation generates demand for precautionary liquidity. This liquidity demand is *excess* in the following senses. First, the entrant would not need liquidity on the time of entry, if there were no threat of predation: he could pay capacity cost by a short-term loan without any precautionary liquidity . Second, this precautionary liquidity is not spent in equilibrium outcome, because there is no predation eventually.

The entrant gets a long-term loan to raise precautionary liquidity on the time of entry. We consider a standard financial contract for this loan and here unverifiability plays the role. The financial contract is designed to satisfy the entrant's limited liability constraint, the lender's participation constraint, and the incentive compatibility for truth telling. The incumbent's capacity size is unverifiable; the lender needs the entrant's report to confirm his actual profit and liquidity holding at the moment of repayment.

Assuming common knowledge, we can say that the lender could predict the equilibrium profit, which however is not verifiable. So he could punish, i.e., refuse the continuation of business, if the entrant reports a non-equilibrium profit level. But, to prevent predation, the entrant need commit himself to continuing business as long as the incumbent can get net benefit from monopolizing the market. Thus, the lender has to allow such plausible predatory loss.

Then, unverifiability of the incumbent's exact capacity size invites the entrant's opportunism. By falsifying predatory loss, the entrant could get remission of loan repayment because of limited liability. To punish false bankruptcy and keep the incentive compatibility, the entrant is required to mortgage his start-up physical capital. The mortgage value sets an upper bound on borrowable precautionary liquidity and, consequently, on preventable predatory loss. An entrant with small capital thus cannot raise enough liquidity to completely invalidate threat of predation. To avoid predation and thus to reduce the incumbent's net monopolization benefit with insufficient liquidity, such an entrant shrinks its capacity size. Therefore, unverifiability of predation distorts the outcome in the product market competition.

The modern contract theory has verified possibility that predation occurs even in equilibrium. The seminal paper is Bolton and Scharfstein (1990), followed by Poitevin (1989), Snyder (1996), Fernández-Ruiz (2004), and Khanna and Schroder (2010).[4] In these models, asymmetric information lies in unobservable parameters on the demand/cost structure,[5] and the entrant's needs for external financing comes from fixed production costs. This theory would fit well with large industries that need large investment for R&D, such as innovative semiconductor (CPU etc.) and telecommunication.[6]

But allegations of predatory pricing are often made from owners of small businesses.[7] Small businesses, such as local retailers, restaurants, and food manufacturers, do not involve large physical uncertainty or large investment.[8] We prove that threat of predation distorts market outcomes even without physical uncertainty or fixed costs, as in such small businesses. Note that realization of predation is eventually avoided in our equilibrium. But we should not negate such possibility; for example if we added exogenous fluctuation to the demand and cost structure, predation might evolve even in equilibrium.

A distinguishable aspect of our financial contract is that unverifiable information is an endogenous variable (capacity size) set by the incumbent, not an exogenous random variable such as the entrant's productivity. We adopt the concept of sequential equilibrium to select sensible outcome; so we select equilibrium outcome by imposing robustness to strategic uncertainty, i.e., perturbation from equilibrium strategies. In this paper, we provide a new

[4] Snyder (1996) introduces renegotiation, and Khanna and Schroder (2010) allow variable output/price levels in Bolton and Scharfstein (1990)'s model. Fernández-Ruiz (2004) is a version of adverse selection. Poitevin (1989) constructs a different model from these and investigates the entrant's choice between equity and debt finances in a one-shot game, as well as introducing variable output level. Since Poitevin (1989) is closest to ours, we provide detailed comparison with ours later in this section.

[5] Actually large physical uncertainty is needed for the conclusion in Bolton and Scharfstein (1990). If the entrant surely achieves high profit under no predation and surely suffers low profit under predation, he is invested with the optimal contract in their model and enters the market. And, once he enters, the entrant can continue his business in the second period: there is no predation.

[6] Fumagali and Motta (2010) provide a succinct summary of EU antitrust litigations in these industries to support their theory.

[7] So are the classic famous lawsuits of predatory pricing: Utah Pie Co. v. Continental Baking Co., 386 U.S. 685 (1967); William Inglis & Sons Baking Co. v. ITT Continental Baking Co., 688 F. 2d 1014 (9th Cir. 1981), *cert. denied*, 459 U.S. 825 (1982); A. A. Poultry Firms, Inc. v. Rose Acre Firms, Inc., 881 F. 2d 1396 (7th Cir. 1989), *cert. denied*, 494 U.S. 1019 (1990).

[8] The success of such local business depends mainly on how well the owner knows the local market and maintains his business, rather than making costly and risky innovation. Taylor and Archer (1994) suggest ten principles and 273 *Kaizen* (improving) suggestions for a local retailer competing against giant supermarkets such as Walmart. The basic message there is to know the business environment, to keep good relationship with customers and to improve the management on a daily basis. It is noteworthy that their banking strategies are to keep and share financial and business information with bankers and to help them to monitor the business, as well as to arrange for credit lines before needing money but not to borrow up the lines.

kind of revelation principle for sequential equilibrium, though it is proven specifically for our model.

The paper proceeds as follows. In the rest of this section, we compare our model with the most relevant preceding literature on financial theory of predation and on revelation principle. The next section describes the economy. Section 3 presents the benchmark where the entrant, as well as the incumbent, does not face the cash-in-advance constraint. In Section 4, we see the case where the rival's capacity is verifiable and present formally threat of predation and the entrant's demand for precautionary liquidity. In Section 5, we show that the unverifiablity results in distortion in the product market. We find that under a version of a direct mechanism, the threat of predation prevents a low-capitalized entrant from raising sufficient excess liquidity (Section 5.1) and forces him to shrink his capacity (Section 5.2); finally, we generalize these results to arbitrary forms of the financial contract by presenting the revelation principle (Section 5.3). In Section 6 we discuss structural assumptions in the model and in the propositions, which leads us to policy and practical implication in the concluding remarks. Some lengthy proofs and the formal presentation of the revelation principle are given in Appendix.

More on the related literature

Among the preceding literature on predation due to financial imperfection, Poitevin (1989) presents predatory excess supply and excess liquidity as a solution of an adverse selection problem. In his model, the entrant chooses either debt or equity to finance liquidity. Excess liquidity is raised by debt, which increases risk of bankruptcy and *stimulates* the incumbent's predation. This is what a high-productivity entrant himself wants. He raises the debt level so high that a low-productivity entrant cannot bear intensified predation; so large debt is a signal of high productivity. In contrast, the incumbent whose productivity is known publicly does not need such a signal and finances his fixed cost by equity. This enables him to exercise predation free from risk of his own bankruptcy. Poitevin sees excess liquidity as a signal of the entrant's productivity, *not as a barrier to predation.*

Our model is a game of complete but imperfect information as there is no uncertainty in the cost and demand structure, while the preceding models of financial predation *à la* Bolton and Scharfstein (1990) involve signaling about the entrant's hidden productivity or demand and thus consider games of incomplete information. Recently some authors construct (nonfinancial) theories of predation under perfect and complete information: see

Argenton (2010) and Fumagali and Motta (2010). Besides, Roth (1996) presents predation as rationalizable strategy (in the sense of Bernheim and Pearce) in War of Attrition. These consider repeated games with sequential buyers, without financial contracting. Our model can be seen as another attempt to formalize a (financial) theory of predation in complete information games.

The standard version of revelation principle considers correlated equilibrium or Bayesian Nash equilibrium.[9] Bester and Strausz (2001) prove a version for perfect Bayesian equilibrium with renegotiation of a contract. In these versions, every type (every possible value of unverifiable information) realizes with exogenous positive probability.

In contrast, the hidden information in our model is the choice of an outsider of the contract (the incumbent's capacity), which is endogenously determined unlike a payoff type in an incomplete information game. Furthermore, we consider perturbations to strategies so as to capture the effect of off-path predation on equilibrium outcomes.

Gerardi and Myerson (2007) employ sequential equilibrium to analyze a Bayesian communication game with a non-full support on the type space. In their model, the agents' *actions* are trembled while the type distribution is fixed. One might interpret the incumbent's capacity (an unverifiable variable that is chosen out of this communication game) as the 'type' of the entrant in their communication game. Yet, their perturbation is essentially different from ours, since we want to tremble the incumbent's capacity (the entrant's *type* in this interpretation). Besides, while every player sends own message in their communication game, the incumbent in our model is out of the contracting party and does not send any message. Hence we cannot apply their model to our situation, i.e., strategic uncertainty controlled by a player who is not bound by a contract.

2 The Economy

We consider an entrant (firm 1) and an incumbent (firm 2) who compete in a product market. Before finishing the production and selling products in the market, each firm makes some precommitment that defines competence in the product market, e.g. a capacity of production, volume of advertisement. The entrant has to pay these costs before completing the production and achieving the sales, because he is new to this business and thus has no reputation to defer the payment unconditionally.

[9] See Osborne and Rubinstein (1994, Proposition 47.1) for the former and Fudenberg and Tirole (1991, Section 7.2) for the latter.

So the entrant faces the cash-in-advance constraint to continue the production and stay in the market. In principle, he can borrow these costs by putting his start-up asset as collateral when he enters the market. After both firms make these commitments, the entrant can borrow an additional loan leveraged by anticipated profit.

Below we formalize the model. The economy starts in period 0 and ends in period 4. Here we separately describe the product market and the financial structure of the entrant. In Fig.1, we summarize all the events in this model, sorting them by time.

The product market

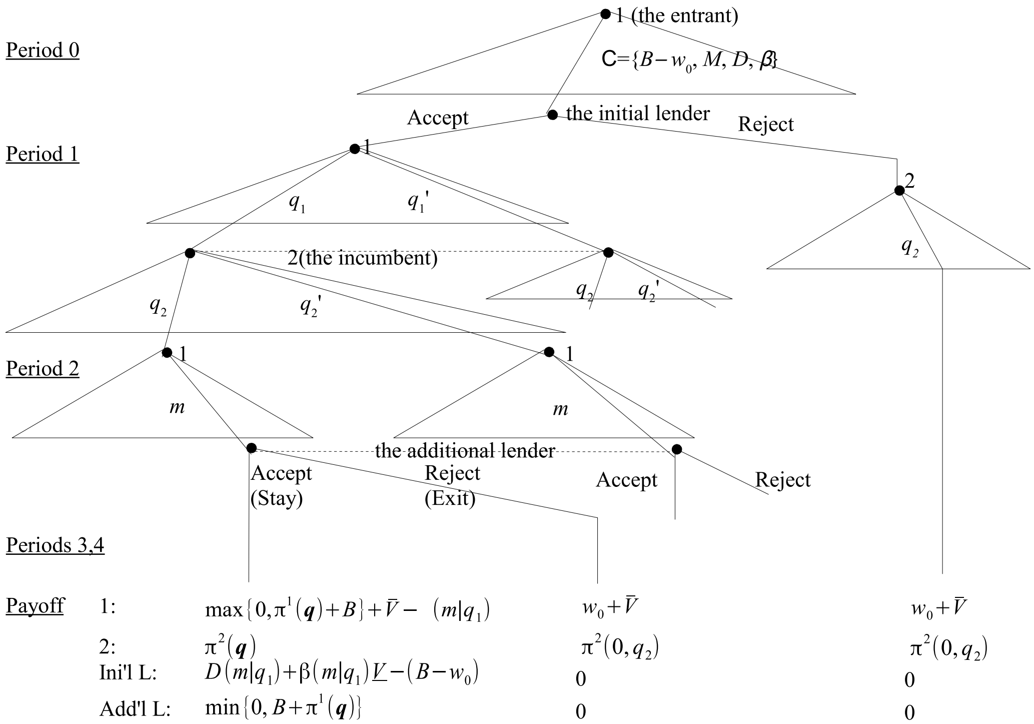
In period 1, each firm $i = 1, 2$ commits to a strategy, say 'capacity size', $q_i \in Q_i \subset \mathbb{R}_+$.[10] Let $Q := Q_1 \times Q_2$ and $\mathbf{q} := (q_1, q_2) \in Q$. We assume that the entrant's capacity q_1 is verifiable in the court by the entrant himself or by the lenders. In contrast, the incumbent's capacity q_2 may or may not be verifiable by the entrant's side; we consider both cases.

In period 2, the entrant must pay out the capacity cost $C^1(\mathbf{q})$ (**the cash-in-advance constraint**), while the "long-purse" incumbent can postpone paying $C^2(\mathbf{q})$ until achieving his sales. If the entrant does not have enough money, he is forced to exit the market.

In period 3, both firms produce and sell the products. We reduce the outcome in the period-3 market competition into a revenue function $R^i : \mathbb{R}_+^2 \to \mathbb{R}$. If the entrant stays in the market, each firm i achieves the revenue $R^i(\mathbf{q})$. If the entrant exits, only the incumbent achieves the revenue $R^2(0, q_2)$.

We denote by $\pi^i(\mathbf{q})$ the firm i's net operating profit: $\pi^i(\mathbf{q}) = R^i(\mathbf{q}) - C^i(\mathbf{q})$. We assume that each $\pi^i : \mathbb{R}_+^2 \to \mathbb{R}$ is continuously differentiable and concave, i.e., $\pi^i_{ii} < 0, \pi^i_{jj} \leq 0$, as well as $\pi^1(0, q_2) = 0, \pi^2(q_1, 0) = 0, \pi^i_j < 0, \pi^i_{ij} < 0$ for each $i = 1, 2$ and $j \neq i$. (Here $\pi^i_j := \partial \pi^i / \partial q_j, \pi^i_{ij} := \partial^2 \pi^i / \partial q_i \partial q_j$.) Hence we assume that larger capacity q_i decreases the rival's net profit π^j ($j \neq i$). Though the source of such substitutability may come from the factor market (increasing the capacity cost C^j) and/or from the output market (decreasing the revenue R^j), our argument applies to both cases. These functions C, R and π are also verifiable and common knowledge. So *the actual revenue is verifiable if and only if the incumbent's capacity is verifiable.*

[10] Although we call q_i the capacity of firm i, it indeed should be seen as a summary variable of all strategies committed before achieving sales, e.g. capacity size, quantity of product that takes so long time to produce, the volume of advertisement. So it is natural that they are unverifiable to outsiders.



Period 0. The entrant with start-up capital w_0 borrows an *initial loan* $B - w_0$ under the financial contract $\mathcal{C}$. B is the entrant's precautionary liquidity at the end of this period.

Period 1. The entrant and the incumbent simultaneously *determine their capacities* $\mathbf{q}$. The entrant's capacity q_1 is verifiable. The incumbent's capacity q_2 is assumed to be verifiable in Section 3 and 4, and to be unverifiable in Section 5.

Period 2. The entrant announces a message $m \in M$ if q_2 is unverifiable. According to the initial loan contract, the amount of monetary repayment $D(m|q_1)$ and the liquidation policy $\beta(m|q_1)$ are determined. The entrant must pay his capacity cost $C^1(\mathbf{q})$ to accomplish the production. If he cannot, he has to give up the production due to the cash-in-advance constraint. To pay this capacity cost, the entrant may borrow *an additional loan* in this period.

Period 3. The entrant (if he stays in the market) and the incumbent sell their products and achieve revenues R^1 and R^2. Then the entrant repays the additional loan.

Period 4. The entrant repays $D(m|q_1)$ to the initial lender. Besides, the initial lender gains the liquidation value $\beta(m|q_1)\underline{V}$. The entrant retains the private continuation value $(1 - \beta(m|q_1))\bar{V}$.

Figure 1: The game tree when the incumbent's capacity is unverifiable and the additional loan is not contracted in period 0. When it is verifiable, the two nodes of the additional lender after ($\mathcal{C}$, Accept, q_1, q_2, m) and ($\mathcal{C}$, Accept, q_1, q_2', m) are separated.

The timings of the entrant's financing

In period 0, the entrant appears in the market with the start-up liquidity $w_0 \geq 0$. The entrant borrows an **initial loan** from a lender. Denote by B the total liquidity holding at the end of period 0 (the **precautionary liquidity**), i.e., the start-up capital w_0 plus the initial loan $B - w_0$.

In period 2, the entrant faces the cash-in-advance constraint. As the precautionary liquidity B may not cover the capacity cost $C^1(\mathbf{q})$, the entrant can ask for an **additional loan**. As a base model, we assume that the additional lender is different from the initial lender and does not commit to additional lending (an uncommitted additional loan). But all the our propositions hold even if the additional loan is prohibited (no additional loan) or committed by the initial lender (credit line). We consider these two cases as supplement to the base model. Anyway, if agreed, the additional loan covers the difference between the precautionary liquidity holding and the capacity costs.

After the product is sold and the entrant achieves revenue R^1, the loans are repaid. We assume that the additional loan is repaid in period 3 before the repayment of the initial loan in period 4, as expressed later. If the liquidity holding at the beginning of period 3 cannot cover the additional loan, the additional loan cannot be fully repaid and the lender loses her money. Otherwise, we assume that she achieves zero profit, imagining competitive market for additional loans to simplify her decision.

We focus on equilibria where the additional lender employs a pure strategy about whether or not to lend the additional lending (contingent on the verifiable information in period 2). Corresponding to this, we assume that the policy of credit line is deterministic. That is, given the verifiable information in period 2, the equilibrium probability to lend the additional loan is either 1 or 0 both in the case of an uncommitted additional loan and in the case of credit line. As this is indeed a crucial assumption especially in the later case, we will discuss later the case of stochastic commitment to additional lending. But, here we note that it would be practically hard to commit to a probability distribution and to make such a stochastic commitment known to an outsider of the contracting party, i.e., the rival (the incumbent) in the product market.

We look at a pure-strategy equilibrium where the initial loan contract is accepted, the additional loan is also approved if necessary for the entrant, and the entrant stays in the market. We call it a **non-predatory equilibrium**. To be approved, the initial loan must be fully repaid in the equilibrium. It is the initial lender's participation condition.

Unverifiability and the contract

The incumbent's capacity size q_2 and consequently the entrant's actual profit may be verifiable or unverifiable in our model. If they are verifiable, full repayment of loans can be enforced by the court as long as the entrant actually has enough liquidity.

In the unverifiable case, the entrant's actual liquidity holding is not verifiable and thus the court cannot enforce the entrant to repay the loan. To create the incentive for him to voluntarily repay it, we assume that the entrant has a non-monetary asset V on the time of entry and put a mortgage on it to borrow the initial loan. As a standard financial contract, the initial loan contract $\mathcal{C}$ consists of the followings.

- $B - w_0 \in \mathbb{R}$: the amount of the initial loan.
- M: the set of available messages that can be sent in the beginning of period 2. We allow M to vary with q_1.[11]
- $D(\cdot|\cdot) : M \otimes Q_1 \to \mathbb{R}$:[12] the (monetary) repayment in period 4 if the entrant stays in the market, given the message $m \in M$ and the entrant's capacity $q_1 \in Q_1$.
- $\beta(\cdot|\cdot) : M \otimes Q_1 \to [0, 1]$: the liquidation policy, i.e., the proportion of the mortgaged asset V that the lender takes over in the end of period 4. We assume that the asset is divisible. That is, the proportion β can take any value in $[0, 1]$, not only $\{0, 1\}$.

The exit is assumed to be verifiable. Then it is verifiable that the entrant does not spend money and thus still holds all the precautionary liquidity B; thus the full repayment is enforceable. So we simply let the entrant repay all the initial loan $B - w_0$ without liquidation of the asset V if he exits from the market. We assume that the contract $\mathcal{C}$ is made public and thus common knowledge for everyone in the economy, as well as it is verifiable in the court.

At the beginning of period 2, the entrant announces the message $m \in M$, after observing the capacities q_1 and q_2 committed in period 1. We assume that the additional lender shares the same message m with the initial lender, which reduces the information problem in the additional lending.

[11] For an arbitrary mechanism, we could think of the union of the message spaces s.t. $M = \cup_{q_1 \in Q_1} M(q_1)$. But we need to explicitly state the dependency of M on q_1 when we discuss a quasi-direct mechanism.

[12] Here $M \otimes Q_1 := \{(m, q_1) | q_1 \in Q_1, m \in M(q_1)\}$.

As the initial lender takes the start-up asset as collateral, we assume that the additional lender has priority to be repaid from the sales of the products.[13] In period 3, the additional loan is repaid right after the entrant achieves the revenue $R^1(\mathbf{q})$. Notice that after the additional loan is wholly repaid, the entrant has liquidity as much as $\pi^1(\mathbf{q}) + B$ in the end of period 3.

In period 4, the initial loan is repaid according to the repayment schedule D. Besides, the initial lender liquidates the proportion $\beta(m|q_1) \in [0, 1]$ of the mortgaged asset and gains the liquidation value $\beta(m|q_1)\underline{V}$. The initial lender's participation condition means in this case that the equilibrium repayment plus the liquidation value should cover the loan $B - w_0$. The entrant retains the rest of the asset and gains the private continuation value $(1 - \beta(m|q_1))\overline{V}$. We assume $\overline{V} > \underline{V} \geq 0$.[14] In contrast with the monetary repayment D, we define the **total repayment** δ as the monetary repayment D plus the entrant's loss from liquidation of the mortgaged asset: $\delta(m|q_1) := D(m|q_1) + \beta(m|q_1)\bar{V}$.

We emphasize that D should be the actual effective amount, not just the face value, of the repayment. Consider a direct mechanism where the entrant announces the incumbent's capacity size. When the entrant reports $m = \tilde{q}_2$ as the incumbent's capacity in period 2 and continues the production, this report implies that the entrant's liquidity holding is $\pi^1(\tilde{q}_2|q_1) + B$ in the beginning of period 4.[15] If the face value of the repayment exceeds it, it cannot be fully paid and the actual repayment is reduced to be within this liquidity holding. This is **the entrant's limited liability constraint**. The initial loan contract should be **valid** in the sense that it satisfies the limited liability and the initial lender's participation condition.

In general, the message space M may be different from Q_2. But in Section 5.3, we see that an outcome from any contract (mechanism) $\mathcal{C}$ is also obtained from a "quasi-direct" mechanism $\hat{\mathcal{C}}$, where given q_1 the entrant announces the rival's capacity q_2 if he wants to stay, or otherwise a message m that prevents him from obtaining the additional loan and lets him exit.

[13] Although we just assume this financial structure, this is realistic as we imagine the following situation: the entrant puts up physical assets to start the business as collateral for an initial loan, and inventories and accounts receivable for an additional loan. See Hart (1995, p.111).

[14] Here we assume that the liquidation value (the continuation value, resp.) is linear in the proportion of the asset that the lender (the entrant, resp.) takes over in period 4. But all of our propositions, esp. the non-predation condition (13), remain the same as long as its minimum is 0 and its maximum is $\underline{V}$ ($\overline{V}$, resp.)

[15] If he gives up the production and exits from the market, the limited liability does not matter, because it is verifiable that the entrant exits and nothing is spent from B.

3 Benchmark: No Cash-in-advance Constraint

As a benchmark, we consider the entrant with no cash-in-advance constraint. That is, the entrant commits to production and he pays capacity costs after he achieves the sales in period 3. We can solve the game like a usual 'Cournot' competition: the benchmark capacity $\mathbf{q}^\dagger$ is determined by

$$q_i^\dagger = \arg\max_{q_i \in Q_i} \pi^i(q_i, q_j^\dagger) \quad \text{for each } i = 1, 2, \ j \neq i. \tag{1}$$

Without the CIA constraint, there is no threat of predation and no need to raise precautionary liquidity on the time of entry. We exclude a trivial case where either firm cannot earn positive profit.

Assumption 1. Assume that $\pi^i(\mathbf{q}^\dagger) > 0$ for each i.

Besides, to characterize an equilibrium by first-order conditions, we assume that the restriction on the capacity space from $\mathbb{R}^2$ to Q does not alter the benchmark equilibrium capacities.

Assumption 2. Let $\hat{\mathbf{q}}^\dagger$ be the solution of (1) when q_i could take any real number: $\hat{q}_i^\dagger = \arg\max_{q_i \in \mathbb{R}} \pi^i(q_i, \hat{q}_j^\dagger)$ for each i. Assume that $\hat{\mathbf{q}}^\dagger \in Q$.

Then the benchmark equilibrium is characterized as

$$\pi_1^1(\mathbf{q}^\dagger) = 0, \quad \pi_2^2(\mathbf{q}^\dagger) = 0. \tag{2}$$

4 Verifiable Case: Excess Liquidity against Predation

Here we see the case where the incumbent's capacity is verifiable but the entrant faces the cash-in-advance constraint in period 2. We specify the notions of "threat of predation" and of "excess liquidity against predation" in our model. Further, we find that the threat of predation does not affect the equilibrium outcome in the verifiable case.

First of all, we clarify that the cash-in-advance constraint alone would not make any difference from the benchmark case, *if the incumbent took the same strategy as the benchmark*: the entrant would not have to raise precautionary liquidity. Suppose that the incumbent never preys on the entrant and takes the benchmark output $q_2^\dagger$, regardless of the entrant's precautionary liquidity holding B: the same strategy as the benchmark case. Without an additional loan, the entrant needed the precautionary liquidity $B \geq C^1(\mathbf{q}^\dagger)$ large enough to meet the cash-in-advance constraint for the capacity cost at $\mathbf{q}^\dagger$. But he has an opportunity to borrow an additional loan. Both firms' capacities $q_1^\dagger$ and $q_2^\dagger$ are assumed here to be verifiable. It is verifiable that the entrant achieves the revenue $R^1(\mathbf{q}^\dagger)$ in period 3. The additional loan is thus available in period 2 if and only if the anticipated revenue $R^1(\mathbf{q}^\dagger)$ and the precautionary liquidity B cover the capacity cost $C^1(\mathbf{q}^\dagger)$: $R^1(\mathbf{q}^\dagger) + B \geq C^1(\mathbf{q}^\dagger)$, i.e., $\pi^1(\mathbf{q}^\dagger) + B \geq 0$. As long as the entry is profitable in the benchmark equilibrium, i.e., $\pi^1(\mathbf{q}^\dagger) > 0$, this condition holds even without precautionary liquidity $B = 0$. So, even if the entrant faces the cash-in-advance constraint, he can stay in the market without any precautionary liquidity, as long as the incumbent unconditionally sets his capacity size to the benchmark $q_2^\dagger$, i.e., if there is no threat of predation.

However, the CIA constraint indeed makes the incumbent's strategy depend on the entrant's precautionary liquidity B and consequently generates the demand for B. As we argued above, the verifiablity guarantees full repayment of the additional loan if and only if the revenue $R^1(\mathbf{q})$ plus the precautionary liquidity B covers the capacity cost $C^1(\mathbf{q})$; then, the entrant can stay in the market, satisfying the CIA constraint by the additional loan. So the condition to stay in the market given $\mathbf{q}$ is

$$R^1(\mathbf{q}) + B \geq C^1(\mathbf{q}), \quad \text{i.e., } \pi^1(\mathbf{q}) + B \geq 0. \tag{3}$$

If this inequality is satisfied, the additional lender is sure and can verify in the court that the additional loan is unspent and that the entrant is able to repay it, and thereby the lender agrees on the loan; otherwise the lender is sure that the additional loan is spent to cover the operating loss and that the entrant is not able to repay it, and thus the lender refuses the loan.[16] So the inequality (3) is the sufficient and necessary condition for the entrant to finance the capacity cost with the additional loan and continue the production in period 2.

We can see the condition (3) as the **liquidity constraint** that the entrant faces in the beginning of period 2. In period 1, each firm determines his own capacity so as to maximize his net profit, the entrant facing the liquidity constraint (3). Given the entrant's

[16] If (3) is not satisfied, the entrant cannot pay the capacity cost from his precautionary liquidity alone.

precautionary liquidity B, the optimal capacity profile $\mathbf{q}^B = (q_1^B, q_2^B)$ is the solution of

$$q_1^B = \arg\max_{q_1 \in Q_1} P(q_1, q_2^B; B)\pi^1(q_1, q_2^B), \tag{4a}$$

$$q_2^B = \arg\max_{q_2 \in Q_2} P(q_1^B, q_2; B)\pi^2(q_1^B, q_2) + (1 - P(q_1^B, q_2; B))\pi^2(0, q_2), \tag{4b}$$

where $P(\mathbf{q}; B)$ is the probability that the entrant gets the additional loan (if necessary) and stays in the market given the capacity profile $\mathbf{q}$ and the precautionary liquidity holding B: it is 1 if the liquidity constraint (3) holds at $\mathbf{q}$ and 0 if not. Here we can see that the incumbent may set a predatory excess capacity because of this liquidity constraint: he can break the liquidity constraint by raising his capacity q_2, which lowers the entrant's anticipated net profit π^1. When he succeeds in such predation, the entrant is forced to exit from the market and the incumbent enjoys the predatory profit $\pi^2(0, q_2)$ by monopolizing the market. So the liquidity constraint brings *threat of predation* to the entrant.

But, the threat of predation is limited as we think of a rational incumbent. As seen in Fig. 2, there is a threshold of the incumbent's capacity $\bar{q}_2^P(q_1)$ where the predatory profit begins to fall below the optimal profit without predation, given the entrant's capacity q_1:

$$Q_2^P(q_1) := \{q_2 \in Q_2 | q_2 \geq q_2^{BR}(q_1), \pi^2(0, q_2) > \pi^2(q_1, q_2^{BR}(q_1))\}, \ \bar{q}_2^P(q_1) := \sup Q_2^P(q_1) \tag{5}$$
$$\text{where } q_i^{BR}(q_j) = \arg\max_{q_i \in Q_i} \pi^i(q_i, q_j).$$

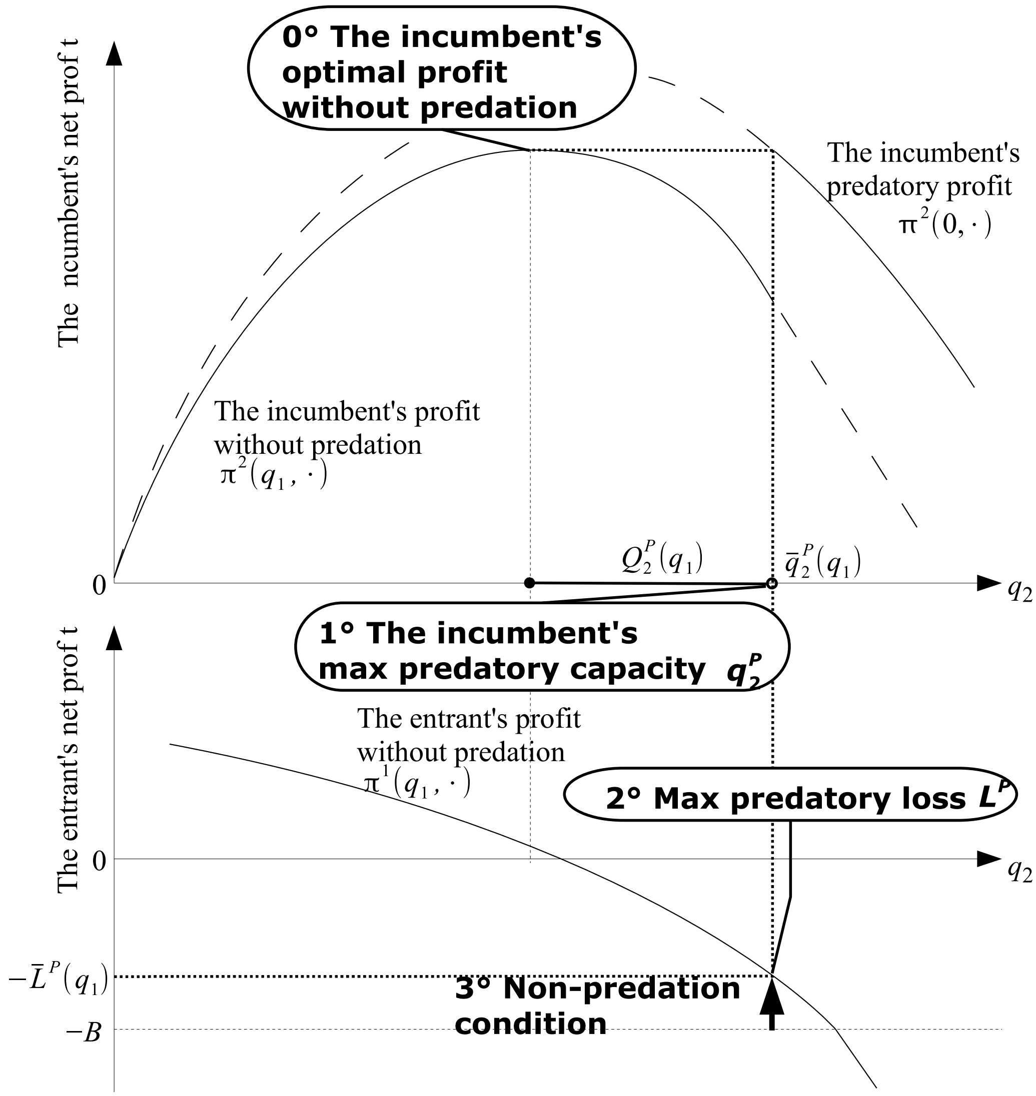
A larger predatory capacity gives the entrant larger operating loss, but a predatory capacity over the threshold $\bar{q}_2^P$ is *implausible* because it makes the incumbent's profit worse than that without predation and the rational incumbent never conducts it. We call the threshold capacity size $\bar{q}_2^P(q_1)$ **the maximal plausible predatory capacity** and the entrant's loss due to this maximal plausible predatory capacity $-\pi^1(q_1, \bar{q}_2^P(q_1))$ **the maximal plausible predatory loss** $\bar{L}^P(q_1)$:

$$\bar{L}^P(q_1) = -\pi^1(q_1, \bar{q}_2^P(q_1)). \tag{6}$$

Setting own capacity size to q_1, the entrant has to be able to continue the production against any plausible predatory capacity $q_2 \in Q_2^P(q_1))$: the liquidity constraint (3) has to be satisfied at $q_2 = \bar{q}_2^P(q_1)$, i.e.,

$$B \geq \bar{L}^P(q_1). \tag{7}$$

If not, the incumbent can enjoy a larger predatory profit $\pi^2(0, q_2)$ than $\pi^2(q_1, q_2^{BR}(q_1))$ by



0° We want to see an equilibrium where the entrant prevents predation; the incumbent's equilibrium capacity maximizes the duopoly profit $\pi^1(q_1, q_2)$ given the entrant's q_1.

1° The incumbent could benefit from predation if and only if the incumbent could get the entrant exit by a predatory capacity less than $\bar{q}_2^P(q_1)$.

2° $\bar{L}^P(q_1)$ is thus the maximum plausible loss of the entrant in case of predation.

3° As long as the entrant can stay in the market even if he suffers the loss of $\bar{L}^P(q_1)$, the incumbent does not prey on him. To guarantee the entrant's stay, the liquidity constraint requires him to possess precautionary liquidity B more than $\bar{L}^P(q_1^0)$.

Figure 2: The maximum plausible predatory loss $\bar{L}^P$ and the non-predation condition given q_1.

setting any predatory capacity $q_2 \in Q_2^P(q_1)$ and obstructing the entrant from borrowing a large enough loan. If this condition (7) is satisfied, the entrant prevents predation and the net profit in period 3 is $\pi^1(\mathbf{q})$ as he planed in period 1. We call the inequality (7) the **non-predation condition under verifiablity**.

The non-predation condition shows that the threat of predation creates the entrant's need for *excess liquidity.* Under no threat of predation, the entrant could finance all the capacity cost $C^1(\mathbf{q})$ by an additional loan and would not have to raise any precautionary liquidity.

Although the non-predation condition could seemingly restrict the entrant's equilibrium capacity, the verifiability of the incumbent's capacity invalidates this restriction. If the non-predation condition is satisfied at equilibrium capacity $q_1^\ddagger$ and the actual predation is totally eliminated, the entrant surely achieves the equilibrium net profit $\pi^1(\mathbf{q}^\ddagger)$ in period 3 and the initial loan is not spent. Since the actual profit is now verifiable, the court can enforce the entrant to repay the whole amount of the initial loan. Anticipating this, the initial lender approves any amount of an loan. The entrant can thus obtain a sufficient initial loan: he can raise B large enough to make the non-predation condition (7) slack.

As a result, the equilibrium capacity profile is the same as the benchmark equilibrium $\mathbf{q}^\dagger$. The only difference is that the entrant needs large enough precautionary liquidity $B^\ddagger \geq \bar{L}^P(q_1^\dagger)$ to meet the non-predation condition (7).

If the additional loan is prohibited, the liquidity constraint (3) is replaced with $B \geq C(\mathbf{q})$ and consequently the non-predation condition is $B \geq C^1(\mathbf{q})$ for any $q_2 \in Q_2^P(q_1^\ddagger)$, tighter than when an uncommitted additional loan is available. If the credit line is available, the liquidity constraint means commitment to lending $C^1(\mathbf{q})$; the non-predation condition is that the credit line covers $C^1(\mathbf{q})$ for any $q_2 \in Q_2^P(q_1^\ddagger)$. Under no threat of predation, the precautionary liquidity or the credit line was required to cover only the actual capacity cost $C^1(\mathbf{q}^\ddagger)$. Hence, similarly to the case of an uncommitted additional loan, the threat of predation makes more demand on the initial loan though it does not distort the capacity sizes in the product market.

Proposition 1. *Consider the case where the entrant faces the cash-in-advance constraint and the rival's capacity q_2 is verifiable.*

1) There is threat of predation: without enough precautionary liquidity on the entrant, the incumbent could exclude the entrant by setting an excess capacity. The entrant needs to raise excess precautionary liquidity on the time of entry so as to prevent the predation, even *if he can borrow an additional loan after the entry.*

2) But, the verifiability enables the entrant to obtain sufficient precautionary liquidity to avoid the incumbent's predation. Consequently, despite of the threat of predation, the equilibrium outcome in the product market is not distorted.

Remark. Notice that we assume zero interest rate on both initial and additional loans. It is obvious that the part 2) in Proposition 1 is not robust if interest is charged for the initial loan: the entrant shrinks the capacity if he needs to pay interest to raise precautionary liquidity. Besides, the extent of distortion depends on the way of financing liquidity against predation: there is no distortion if the credit line does not incur any fee or interest when it is not used.

5 Unverifiable Case: Distortion in the Product Market

In this section we see that need for excess liquidity against threat of predation indeed distorts the product market equilibrium, if the incumbent's capacity is not verifiable. Combining incentive compatibility for truth telling (due to the unverifiability) and limited liability of the entrant, we obtain a non-trivial condition for the entrant to stay in the market; it requires start-up liquidity w_0 plus continuation value $\bar{V}$ of the leveraged asset to be large enough to cover the maximal plausible predatory loss $\bar{L}(q_1)$. Through this non-predation condition, these two parameters restrict the entrant's equilibrium capacity.

5.1 Non-predation condition in a quasi-direct mechanism

In the last section, we see that, to prevent predation, the entrant has to be able to stay in the market and continue the business in period 2 as long as the incumbent's capacity q_2 is less than the maximal plausible predatory capacity $\bar{q}_2^P(q_1)$. Even though it guarantees the same equilibrium output and profit as in the benchmark equilibrium and thus the loans are fully repaid in the equilibrium outcome, this continuation policy allows remittance of the repayment in case of off-equilibrium predation, due to the entrant's limited liability.

As the capacity strategies are determined as (4) is easy to see that such an anti-predation continuation policy is needed in the unverifiable case too. The question here is whether or not such a policy is implementable while keeping the incentive for the entrant to fully repay the loans, when the rival's capacity q_2 and presence/absence of predation are revealed only through the entrant's voluntary report m. For now, we consider a version of a direct mechanism to investigate the relation between the continuation policy against predation and the incentive of false bankruptcy; this dilemma causes restriction on the equilibrium capacities. We will extend this result to a general mechanism by verifying revelation principle later.

We mean specifically by a mechanism the pair of the initial loan contract $\mathcal{C}$ and the continuation policy $a : M \otimes Q_1 \to \{0,1\}$: $a(m|q_1) = 1$ ($a(m|q_1) = 0$, resp.) means that the lenders agree on (reject, resp.) the entrant to stay in the market in period 2 when he sets capacity size to $q_1 \in Q_1$ and sends message $m \in M$. If additional loan is prohibited, a is determined by the amount of precautionary liquidity B: it is whether the B alone covers the capacity cost. In a quasi-direct mechanism where the entrant tells the true q_2, the implied policy a is

$$a(\tilde{q}_2|q_1) \begin{cases} = 1 & \text{if } B > C^1(q_1, \tilde{q}_2), \\ \in [0,1] & \text{if } B = C^1(q_1, \tilde{q}_2), \\ = 0 & \text{if } B < C^1(q_1, \tilde{q}_2). \end{cases}$$

If additional loan is allowed, $a = 1$ means its approval. In the case of a credit line, a is contracted in period 0 together with $\mathcal{C}$. If the additional loan is not committed, a is determined by the additional lender and thus subject to the *additional lender's* incentive compatibility in period 2. Under the entrant's truth telling, the additional lender's incentive compatibility is

$$a(\tilde{q}_2|q_1) = 1 \Longrightarrow \pi^1(q_1, \tilde{q}_2) + B \geq 0. \tag{8}$$

In this subsection, we let the entrant announce directly the unverified information, namely the incumbent's capacity q_2, *as long as he wants to stay in the market.* Otherwise, the exit is verifiable and q_2 is no longer related with the entrant's liquidity holding; so, if he wants to exit, we let him announce only the intent of exit. So the message space here is the set of the incumbent's capacity sizes $Q_2^S(q_1)$ that allow the entrant to stay and the set of the messages $M_0(q_1)$ that imply the intent of exit. (Note that we allow both sets to depend on q_1.) The continuation policy should let the entrant stay if he announces a message in $Q_2^S(q_1)$ and exit if he announces one in $M_0(q_1)$. We call such a mechanism $(\mathcal{C}, a)$ a quasi-direct mechanism.

Definition 1 (quasi-direct mechanism)**.** The pair of the initial loan contract $\mathcal{C} = \{B - w_0, M, D, \beta\}$ and the continuation policy $a : M \otimes Q_1 \to [0,1]$ is a **quasi-direct mechanism**

if

1) for each $q_1 \in Q_1$, the message space $M(q_1)$ contains a subset of Q_2: so it is written as $M(q_1) = Q_2^S(q_1) \cup M_0(q_1)$ with $Q_2^S(q_1) \subset Q_2$ and $Q_2^S(q_1) \cap M_0(q_1) = \emptyset$; and,

2) the additional lender approves an additional loan if the entrant tells any $\tilde{q}_2 \in Q_2^S(q_1)$, and rejects it if he tells any $m \in M_0(q_1)$.

The mechanism $(\mathcal{C}, a)$ is valid if the initial loan contact $\mathcal{C}$ is valid, namely if it satisfies the initial lender's participation condition and the entrant's limited liability.

Here, we look at a non-predatory equilibrium where the anti-predation continuation policy is truthfully implemented under a valid quasi-direct mechanism. This requires four necessary conditions on the quasi-direct mechanism. One is that the continuation policy is anti-predation:

$$q_2^P \in Q_2^S(q_1^*) \quad \text{for any } q_2^P \in Q_2^P(q_1^*), \ \text{ i.e., } Q_2^P(q_1^*) \subset Q_2^S(q_1^*). \tag{9}$$

Second, the truth telling must be compatible with the entrant's incentive. Given that the entrant wants to stay, he can choose any message in $Q_2^S(q_1)$, especially the one that minimizes the total repayment. To guarantee the truth telling, the total repayment of any message in $Q_2^S(q_1)$ should be equal to the minimum:

$$\underline{\delta}(q_1) := \min_{\tilde{q}_2 \in Q_2^S(q_1)} \delta(\tilde{q}_2 | q_1).$$

So the incentive compatibility for the entrant to tell the true information is

$$\delta(\tilde{q}_2 | q_1) = \underline{\delta}(q_1) \quad \text{for any } \tilde{q}_2 \in Q_2^S(q_1). \tag{10}$$

Third, to get the initial loan contract accepted, the initial lender's participation constraint has to be satisfied in the equilibrium outcome. Given the equilibrium capacity sizes $\mathbf{q}^*$, it is

$$D(q_2^* | q_1^*) + \beta(q_2^* | q_1^*) \underline{V} \geq B - w_0. \tag{11}$$

Finally, the monetary repayment schedule in a valid contract has to satisfy the limited liability. In a truth-teling quasi-direct mechanism, the limited liability condition is

$$D(\tilde{q}_2|q_1) \leq \pi^1(q_1, \tilde{q}_2) + B \qquad \text{whenever } \tilde{q}_2 \in Q_2^S(q_1). \tag{12}$$

The next theorem says that these four constraints jointly impose a non-trivial condition (13) on the entrant's equilibrium capacity q_1^*. We look at the entrant's messaging strategy and the continuation policy given the incumbent's equilibrium capacity q_2^* and given plausible predatory capacity $q_2^P \in Q_2^P(q_1^*)$ with the entrant's equilibrium capacity q_1^*. We call the condition (13) **the non-predation condition under the unverifiability of the rival's capacity**. In Section 5.3, we verify generality of this condition under an arbitrary mechanism.

Theorem 1. *Consider a non-predatory equilibrium in a valid quasi-direct mechanism that truthfully implements the anti-predatory continuation policy, or satisfies the conditions* (9)–(12)*, under the unverifiability of the rival's capacity. Then, the entrant's equilibrium capacity* q_1^* *satisfies*

$$\bar{V} + w_0 \geq \bar{L}^P(q_1^*). \tag{13}$$

Proof. First of all, the continuation policy (9) says that any plausible predatory capacity $q_2^P \in Q_2^P(q_1^*)$ should be accepted, i.e., $q_2^P \in Q_2^S(q_1^*)$, as well as the equilibrium capacity q_2^*.

Since $\bar{V} > \underline{V}$ and $\beta \geq 0$, the participation condition (11) sets a lower bound on the total repayment after the equilibrium capacity q_2^* is announced:

$$\delta(q_2^*|q_1^*) = D(q_2^*|q_1^*) + \beta(q_2^*|q_1^*)\bar{V} \geq B - w_0. \tag{14}$$

Since $\beta \in [0, 1]$ and $\bar{V} > 0$, the limited liability (12) sets an upper bound on the total repayment after a plausible predatory capacity q_2^P is announced:

$$\delta(q_2^P|q_1^*) = D(q_2^P|q_1^*) + \beta(q_2^P|q_1^*)\bar{V} \leq \pi^1(q_1^*, q_2^P) + B + \bar{V}. \tag{15}$$

Finally, since $Q_2^P(q_1^*) \subset Q_2^S(q_1^*)$, the incentive compatibility condition (10) implies $\delta(q_2^P|q_1^*) = \underline{\delta}(q_1^*) = \delta(q_2^*|q_1^*)$ for any $q_2^P \in Q_2^P(q_1^*)$. Combining the two bounds (14) and

(15) with this, we obtain

$$\pi^1(q_1^*, q_2^P) + B + \bar{V} \geq \delta(q_2^P|q_1^*) = \delta(q_2^*|q_1^*) \geq B - w_0 \quad \text{for any } q_2^P \in Q_2^P(q_1^*),$$

$$\therefore \bar{V} + w_0 \geq \max\{-\pi^1(q_1^*, q_2^P)|q_2^P \in Q_2^P(q_1^*)\} = \bar{L}^P(q_1^*).$$

□

In the next section, we see how the non-predation condition distorts the equilibrium outcome in the product market. It is clear from the proof above that we can generalize the result from the truth-telling quasi-direct mechanism to an arbitrary (not necessarily truth-telling) quasi-direct mechanism, as long as the mechanism implies the same incentive compatibility condition (10).

It is worth to notice that the proof above does not depend on the way of additional lending and thus neither does the non-predation condition (13). In particular, we have the maximal plausible predatory loss $\bar{L}^P(q_1^*)$, not the capacity cost $C^1(q_1^*, \bar{q}_2^P(q_1^*))$, on the non-predation condition (13) even in the case of no additional loan. This is because it comes from the limited liability constraint on the entrant's liquidity *in period 4* (after paying the capacity cost and repaying the additional loan), not in period 2 (before it).

Recall that when the incumbent's capacity is verifiable, the non-predation condition (7) is not restrictive because of the freedom to raise the precautionary liquidity B by the initial loan; when it is unverifiable, the non-predation condition (13) is restrictive for the entrant with small start-up capital $\bar{V} + w_0$.

While (13) is just a necessary condition on the equilibrium capacity profile, we expect the entrant and the initial lender to design the loan contract $\mathcal{C}$ so as to maximize the entrant's profit within the non-predation condition (13), anticipating the incumbent's optimal decision on the capacity:

$$q_1^* = \arg\max_{q_1 \in Q_1} \left\{\pi^1(q_1, q_2^*) \;\middle|\; \bar{V} + w_0 \geq \bar{L}^P(q_1)\right\}, \tag{16a}$$

$$q_2^* = \arg\max_{q_2 \in Q_2} \pi^2(q_1^*, q_2). \tag{16b}$$

Even though the entrant's decision on capacity size in period 1 is directly restricted by the liquidity constraint on precautionary liquidity as in (4a), the next theorem says that this capacity profile is obtained as the outcome in a sequential equilibrium after the initial loan is approved.

Theorem 2. *In the unverifiable case, there exists a non-predatory (post-entry sequential) equilibrium with the capacity profile* $\mathbf{q}^*$ *such as* (16)*, as long as* $\pi^1(\mathbf{q}^*) \geq 0$.

Proof. Here we prove the theorem only for the case of an uncommitted additional loan; the proof is easily modified for the other cases. Let $Q_1^S := \{q_1 \in Q_1 | \bar{L}^P(q_1) \leq \bar{V} + w_0\}$ and $Q_2^S(q_1) := \{q_2 \in Q_2 | \pi^1(q_1, \tilde{q}_2) \geq -(\bar{V} + w_0)\}$. Note that $Q_2^P(q_1) \subset Q_2^S(q_1)$ if and only if $q_1 \in Q_1^S$. Consider the initial loan contract $\mathcal{C}$ such as $B - w_0 = \bar{V}$; for each $q_1 \in Q_1$

$$M(q_1) = \begin{cases} Q_2^S(q_1) \cup \{m_0\} & \text{if } q_1 \in Q_1^S, \\ \{m_0\} & \text{otherwise,} \end{cases}$$

$$D(m_0|q_1) = B - w_0, \quad \beta(m_0|q_1) = 0;$$

for each $q_1 \in Q_1^S$ and $\tilde{q}_2 \in Q_2^S(q_1)$,

$$D(\tilde{q}_2|q_1) = \begin{cases} B - w_0 & \text{if } \pi^1(q_1, \tilde{q}_2) \geq -w_0, \\ \pi^1(q_1, \tilde{q}_2) + B & \text{if } \pi^1(q_1, \tilde{q}_2) \in [-(\bar{V} + w_0), -w_0], \end{cases}$$

$$\beta(\tilde{q}_2|q_1) = \begin{cases} 0 & \text{if } \pi^1(q_1, \tilde{q}_2) \geq -w_0, \\ -(\pi^1(q_1, \tilde{q}_2) + w_0)/\bar{V} & \text{if } \pi^1(q_1, \tilde{q}_2) \in [-(\bar{V} + w_0), -w_0]. \end{cases}$$

The additional lending strategy (continuation policy) is to let the additional lender approve the additional loan if the entrant' capacity q_1 belongs to Q_1^S and any message in $Q_2^S(q_1)$ is sent and to let her reject the additional loan if the message m_0 is sent. The entrant's messaging strategy is to let him tell the true q_2 as long as the capacity profile $\mathbf{q}$ belongs to $Q_1^S \otimes Q_2^S$ and to let him send the message m_0 otherwise.

See Appendix B for the rest of this proof; there, we verify that these strategies constitute a non-predatory sequential equilibrium with the capacity profile $\mathbf{q}^*$ in the game after the initial loan contract as above is accepted. □

In summary, threat of predation requires the lender to commit to production in the case of plausible predation, though it is never realized in equilibrium. This commitment is also the source of the borrower's opportunism aiming at the remission of the loan by the limited liability. Under the unverifiablity of predation, the entrant could ask for remission of the initial loan as much as the maximal plausible predatory loss $\bar{L}^P(q_1^*)$. Such an opportunism restricts the low-capitalized entrant's financing and equilibrium capacity.

5.2 Distortion in the product market

Because our model specifies the maximal plausible predatory loss as (6), we can evaluate from the reduced form (16) how much the equilibrium capacities are distorted under threat of unverifiable predation.

In this section we determine q_1^*, q_2^* and $\bar{q}_2^P(q_1^*)$ analytically. To justify it, we make the following assumption on the strategy space like Assumption 2:

Assumption 3. Let $\hat{\mathbf{q}}^*$ be the solution of (16) and $\hat{q}_2^P(q_1)$ be the solution of (5) when q_i could take any real number. The capacity space Q is assumed to contain $\hat{\mathbf{q}}^*$ and $\hat{q}_2^P(\hat{q}_1^*)$:

$$\hat{q}_1^* \in Q_1, \qquad \hat{q}_2^*, \hat{q}_2^P(\hat{q}_1^*) \in Q_2.$$

For such Q, the solution $\mathbf{q}^*$ of (16) and the solution $\bar{q}_2^P(q_1^*)$ of (5) at $q_1 = q_1^*$ coincide with these $\hat{\mathbf{q}}^*$ and $\hat{q}_2^P(\hat{q}_1^*)$.

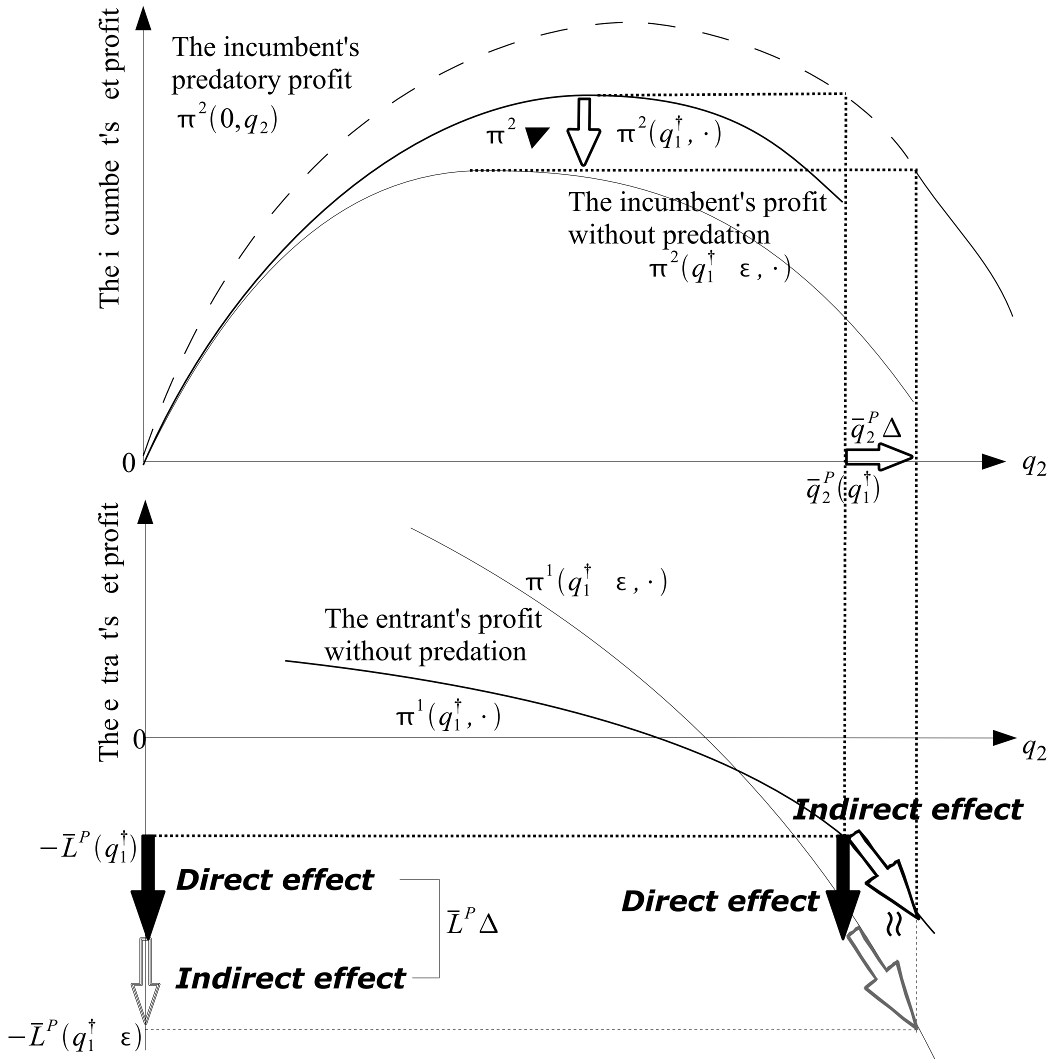
We focus on the entrant who has so small start-up liquidity that violates the non-predation condition at the benchmark capacity size $q_1^\dagger$: namely, assume $w_0 < \bar{L}^P(q_1^\dagger) - \bar{V}$. Otherwise there is no distortion of both firms' capacities. The next corollary of Theorem 1 tells that such a low-capitalized entrant *reduces* his capacity (and the incumbent *increases* his in response) from the benchmark $q_1^\dagger$.

Corollary 1. *Suppose Assumptions 1, 2 and 3. If the entrant's start-up liquidity w_0 is below $\bar{L}^P(q_1^\dagger) - \bar{V}$, the unverifiability of the rival's capacity does not allow the entrant to borrow sufficient precautionary liquidity to avoid predation at the capacity sizes $\mathbf{q}^*$ in the benchmark equilibrium. Consequently, the entrant's capacity shrinks while the incumbent's expands, compared to the benchmark equilibrium $\mathbf{q}^\dagger$.*

In contrast, if w_0 is above $\bar{L}^P(q_1^\dagger) - \bar{V}$, the entrant has no financial difficulty in borrowing precautionary liquidity to avoid predation. Then the equilibrium capacities are not distorted from $\mathbf{q}^\dagger$, like the verifiable case.

Proof. See Appendix C for a formal proof. □

The non-predation condition (13) shrinks the entrant's capacity, as the maximal plausible predatory loss $\bar{L}^P(q_1)$ increases with q_1 at least in a neighborhood of the benchmark $q_1^\dagger$. When the condition (13) is violated at $q_1^\dagger$, the entrant must reduce the maximal plausible



Direct effect Provided that $\bar{q}_2^P$ was unchanged, the entrant's profit would change by the marginal profit due to the increase of q_1. As the marginal profit should be negative at $(q_1^\dagger, \bar{q}_2^P(q_1^\dagger))$, this effect increases $\bar{L}^P$ at $q_1^\dagger$.

Indirect effect As q_1 increases, the incumbent's profit without predation shrinks at any q_2; his net profit of predation gets larger. This allows a larger predatory capacity, i.e., $\bar{q}_2^P$ increases. This also increases $\bar{L}^P$.

Figure 3: Effect of increase in the entrant's capacity q_1 on the maximal plausible predatory loss $\bar{L}^P$ around the benchmark equilibrium $\mathbf{q}^\dagger$.

predatory loss $\bar{L}^P$ by setting smaller q_1. We can decompose the effect of marginal increase in q_1 on $\bar{L}^P$ into direct and indirect effects:

$$\frac{d\bar{L}^P}{dq_1}(q_1) = \underbrace{-\pi_1^1(q_1, \bar{q}_2^P)}_{\text{Direct effect}} \underbrace{-\pi_2^1(q_1, \bar{q}_2^P) \times \frac{d\bar{q}_2^P}{dq_1}(q_1)}_{\text{Indirect effect}},$$

$$\text{where } \frac{d\bar{q}_2^P}{dq_1}(q_1) = \frac{\pi_1^2(q_1, q_2^{BR}(q_1))}{\pi_2^2(0, \bar{q}_2^P(q_1))} > 0.$$

The direct effect is the increase in $\bar{L}^P$, with the incumbent's capacity fixed, caused by the increase in the entrant's capacity itself; the indirect one represents the increase caused by the change in the incumbent's maximal plausible predatory capacity $\bar{q}_1$.

First, the direct effect is positive at the entrant's benchmark capacity size $q_1^\dagger$. Since $\pi_{12}^1 < 0$ and $\bar{q}_2^P(q_1^\dagger) > q_2^\dagger$ (by $\pi_1^2 < 0$ and $\pi_{22}^2 < 0$), the predation $\bar{q}_2^P(q_1^\dagger)$ decreases the entrant's marginal net profit π_1^1 from that at the benchmark equilibrium, $\pi_1^1(\mathbf{q}^\dagger) = 0$. Hence we have

$$[\text{Direct effect at } q_1^\dagger] = -\pi_1^1(q_1^\dagger, \bar{q}_2^P) \geq -\pi_1^1(\mathbf{q}^\dagger) = 0.$$

The indirect effect is also positive if $d\bar{q}_2^P/dq_1$ is positive, as $\pi_2^1(\cdot) < 0$. Positive $d\bar{q}_2^P/dq_1$ means that increase of the entrant's capacity allows the incumbent to be still better off by predation with a larger excess capacity. This is always the case. Increase in the entrant's capacity size q_1 decreases the incumbent's profit without predation $\pi^2(q_1, q_2^{BR}(q_1))$ by $\pi_1^2 < 0$, while his (would-be) predatory profit $\pi^2(0, q_2^P)$ remains the same for any predatory capacity q_2^P. Accordingly, as the entrant increases his capacity q_1, the incumbent's net benefit of the predation becomes larger, and thus the maximal plausible predatory capacity $\bar{q}_2^P$ expands: namely, $d\bar{q}_2^P/dq_1 > 0$.

Because both the direct and the indirect effects are positive, the maximal plausible predatory loss $\bar{L}^P$ increases with the entrant's capacity; thus the low-capitalized entrant should reduce his capacity from the benchmark one $q_1^\dagger$ to meet the non-predation condition (13). Hence, unless the entrant has large enough capital to satisfy the non-predation condition at the benchmark equilibrium, i.e., $w_0 > \bar{L}^P(q_1) - \bar{V}$, the entrant's optimal response to q_2 shifts downward at least around the benchmark equilibrium, while the incumbent's remains the same as the benchmark. That is, threat of predation makes the entrant less aggressive.

Social inefficiency under threat of predation Political/legal intervention on predatory conducts can be justified if the threat of unverifiable predation reduces social welfare, in addition to distorting the product market outcome. Since we work on a very general demand and cost structure, our model may have both positive and negative results: in general the social welfare may and may not decrease under threat of predation.

Yet we have one concrete case where the social welfare decreases.

Corollary 2. *Suppose Assumptions 1, 2 and 3. Furthermore, assume that the capacity cost function is linear in the own capacity, i.e.,* $C^i(\mathbf{q}) = c_i q_i$ *and the revenue function is linear respectively in the total capacity and in the own capacity, i.e.,* $R^i(\mathbf{q}) = \{\alpha - (q_1 + q_2)\}q_i$ *with sufficiently high demand level* α *compared to the entrant's unit cost* c_1*:* $\alpha/c_1 > -7 + \sqrt{66} \approx 1.12$*. If the two firms' productivities* c_1 *and* c_2 *are close enough, then the maximal plausible predatory loss at the benchmark capacity* $\bar{L}^P(q_1^\dagger)$ *increases with the incumbent's unit cost* c_2*. That is, as the incumbent has less efficient technology, the low-capitalized entrant is more likely to shrink his capacity under threat of unverifiable predation.*

Proof. Calculate $\bar{L}^P(q_1^\dagger)$ in this case at $c_1 = c_2$, and differentiate it with regard to c_2. Then we obtain $d\bar{L}^P(q_1^\dagger)/dc_2 > 0$ □

In this case, given the total capacity, it is socially inefficient that the entrant sets a capacity smaller than the incumbent, as both two firms have linear production technology and the entrant has better one. Hence, the equilibrium capacity under threat of unverifiable predation is more socially inefficient than the benchmark.

Product market environment to intensify threat of predation The low-capitalized entrant reduces his capacity when the maximal plausible predatory loss is so huge that the non-predation condition is violated at the benchmark. Here we make a list of the situations where the maximal plausible predatory loss becomes large.

As seen in Corollary 1, the maximal plausible predatory loss increases with the entrant's capacity. The entrant's benchmark capacity gets larger if he has *better productivity* in building a capacity (less c_1) or the demand for the entrant's product has *lower price elasticity* (larger $|R_1^1|$).

Next, consider the situation where the incumbent can reduce the entrant's revenue much from the benchmark $R^1(\cdot, q_2^\dagger)$ by a small predatory capacity. This happens if the entrant's product is little differentiated from the incumbent's and has *high substitutability* (large $|R_2^1|$).

5.3 Revelation principle under strategic uncertainty

In the last section, we see that the non-predation condition causes distortion of the equilibrium outcome in the product market, presuming a truthful implementation of anti-predatory continuation policy in a quasi-direct mechanism. Yet the entrant and the initial lender might try to write a better contract so as to prevent the entrant from the opportunism. So we need to think a broader range of possible contracts for robustness of the non-predation condition.

Here we prove the revelation principle: any outcome under an arbitrary contract reduces to an outcome under a quasi-direct mechanism. Several versions of the principle are already proved and widely used for a variety of equilibrium concepts and games: a correlated equilibrium under perfect information, a Bayesian Nash equilibrium under incomplete information, and a perfect Bayesian equilibrium under incomplete information without the principal's perfect commitment (Bester and Strausz, 2001). Unlike incomplete information games, the content of the unverified information q_2 is determined endogenously by the outsider of the contract—the incumbent, not exogenously by the nature, and almost all alternatives in Q_2 do not realize on a pure-strategy equilibrium path. In our problem, off-equilibrium paths however play a crucial role, because we want to see the effect of *threat* of predation on the product market in a *non-predatory* equilibrium.

Thus we adopt the concept of *sequential equilibrium* to select a sensible outcome after the contract is accepted. This means in essence that we select the pair of strategy and belief, if 1) the strategy is optimal given the belief and 2) the belief is stable when the strategy is perturbed (in a specific way) to take every action with a positive small probability. This perturbation determines the belief especially on the off-equilibrium path by Bayes rule.

Stability under strategy perturbation can be seen as robustness to strategic uncertainty. While the players anticipate the opponents' actions correctly on the equilibrium path, the perturbation further requires an off-path belief to be robust to uncertainty in the opponents' strategy that is caused by the perturbation.

If we stick to a general—possibly continuous—strategy space, it is hard to define a sequential equilibrium and to prove its existence. To our knowledge, the existence in a continuous strategy space is established for a trembling-hand perfect equilibrium in a normal form (Méndez-Naya, García-Jurabo, and Cesco, 1995), but not yet for a sequential equilibrium in an extensive form. So we assume a finite strategy space. This would not be crucial for our analysis, because as we see in Section 5.1, only $\bar{q}_2^P(q_1^*)$ and q_2^* are decisive capacity sizes to induce the non-predation condition and the distortion in the product market.

We start the proof of our revelation principle from categorizing messaging strategies in equilibrium. In general, the message space $M(q_1)$ can vary with q_1, which is observable for the lenders. We focus on equilibrium where each message lets the entrant either stay in the market surely or exit surely.

We first categorize messages in $M(q_1)$ that are sent with positive probability at some $q_2 \in Q_2$. For each q_1, denote by $M_0(q_1) \subset M(q_1)$ the set of such possibly sent messages that let the entrant exit surely, and by $M_1(q_1) \subset M(q_1) \setminus M_0(q_1)$ the set of possibly sent messages that let the entrant stay surely.

1) a pooling-stay case $M_0(q_1) = \emptyset$: the entrant stays for any q_2, i.e., $Q_2^S(q_1) = Q_2$;

2) a pooling-exit case $M_1(q_1) = \emptyset$: the entrant exits for any q_2, i.e., $Q_2^S(q_1) = \emptyset$;

3) a separating case $M_0(q_1), M_1(q_1) \neq \emptyset$: stay or exit depends on q_2, i.e., $\emptyset \neq Q_2^S(q_1) \subsetneq Q_2$.

Here again, $Q_2^S(q_1)$ is the set of the incumbent's capacity size $q_2 \in Q_2$ given which the entrant sends a message that induces approval of additional loan with positive probability. As long as the entrant is expected to stay in the market at such q_2, the monetary repayment D should be designed to be within his liquidity holding after achieving $\pi^1(q_1, q_2)$. It is the *limited liability constraint in a general mechanism.*

Focusing on sequential equilibria in the game after the initial loan contract is accepted, we can convert any mechanism with an arbitrary message space M to a quasi-direct mechanism (not necessarily inducing truth telling) with $\hat{M}(\cdot) = Q_2^S(\cdot) \cup M_0(\cdot)$, while preserving the equilibrium outcome. This is our revelation principle (restated more formally in Appendix A), which is summarized as follows.

Theorem 3 (Revelation principle)**.** *Suppose that the strategy space is (arbitrarily) finite and $w_0 > 0$. Consider a non-predatory sequential equilibrium of the game after the contract is accepted. We can convert a mechanism with the message space for each q_1 to either one of quasi-direct mechanisms below, keeping the same capacity strategies and the same probability that entrant stays in the market after each possible $\mathbf{q} \in Q_1 \times Q_2$:*

1) *A pooling-stay case reduces to a* **pooling-stay mechanism***: the message space $\hat{M}(q_1)$ is the whole Q_2, and the entrant sends each message $\tilde{q}_2 \in Q_2$ with equal probability*

regardless of the actual q_2. The posterior belief is the same as the prior, i.e., the incumbent's strategy of q_2, regardless of the message $\tilde{q}_2$. The net repayment $\delta(\tilde{q}_2|q_1)$ must be constant among all $\tilde{q}_2 \in Q_2$, given $q_1 \in Q_1$.

2) *A pooling-exit case reduces to a* **pooling-exit mechanism***: the message space $\hat{M}(q_1)$ is the same as the original mechanism. The entrant's strategy of m and the posterior belief are the same as the original mechanism.*

3) *A separating case reduces to a* **truth-telling separating mechanism** *with $\hat{M}(q_1) = Q_2^S(q_1) \cup M_0(q_1)$, where $Q_2^S(q_1) \neq \emptyset$ and $M_0(q_1) \neq \emptyset$ are of the original equilibrium. The anti-predatory continuation policy is truthfully implemented.*

Proof. See Appendix A for the proof and the detail of this theorem, as well as the details of Lemma 1 that justifies the above categorization. □

As long as the incumbent capacity size q_2 is in the set $Q_2^S(q_1)$, the entrant is allowed to continue the business given the capacities (q_1, q_2). For the contract in a quasi-direct mechanism to be valid, the limited liability constraint takes the form of (12) and any sent message $\tilde{q}_2 \in Q_2^S(q_1)$ induces a constant net repayment (see (19) in Appendix A), whether it is the truth-telling separating mechanism or the pooling-entry mechanism. So we obtain the same non-predation condition for a non-predatory equilibrium (thus excluding pooling-exit cases) by reducing an arbitrary contract to a quasi-direct mechanism and using Theorem 1.

Theorem 4. *Suppose that the strategy space is (arbitrarily) finite and $w_0 > 0$. Consider a non-predatory sequential equilibrium of the game after a valid initial loan contract is accepted. Under the unverifiability of the rival's capacity, the entrant's equilibrium capacity q_1^* must satisfy the non-predation condition* (13)*.*

Proof. Combine the revelation principle (Theorem 3) with Theorem 1. □

The non-predation condition becomes stronger in the pooling-stay case than the separating case and thus it restricts the monetary repayment more tightly. Even if the incumbent's capacity is above $\bar{q}_2^P(q_1^*)$, the message cannot convey this information in a pooling-stay equilibrium and consequently the entrant is allowed to stay. Thus both the limited liability constraint (12) and the incentive compatibility (10) must hold for any $q_2 \in Q_2$, not only for $q_2 \in Q_2^P(q_1^*)$. So the non-predation condition (13) gets stronger, altered with $\bar{V} + w_0 \geq -\pi^1(q_1, \bar{q}_2)$ where $\bar{q}_2$ is the maximum of Q_2.

In summary, in a non-predation equilibrium, the entrant's capacity q_1 is restricted by his start-up capital w_0 and $\bar{V}$ through the non-predation condition, as long as the loan contract needs to be robust to strategic uncertainty.

Proposition 2. *Consider the case where the entrant faces the cash-in-advance constraint and the rival's capacity q_2 is not verifiable. Assume that the strategy space is (arbitrarily) finite and the liquidity holding on the time of entry is positive*

1) There is threat of predation, and thus the entrant needs to raise excess precautionary liquidity to avoid the predation.

2) Thanks to the unverifiability, the entrant with small start-up capital cannot finance precautionary liquidity large enough to keep the benchmark capacity. With Assumptions 1, 2 and 3, this implies that the entrant's capacity shrinks compared to the benchmark.

6 Discussion

In this section, we discuss the structural assumptions underlying in our model and in our revelation principle for the unverifiable case. The discussion below clarifies applicability of each proposition and guides us to policy implication that we shall make in the next section.

Commitment to 'capacity'

We assume that each firm i commits himself to q_i. While we name it 'capacity', the variable q_i can be anything that determines competence in the subsequent product market. We can think $R^i(\mathbf{q})$ as the *reduced form* of firm i's profit in the product market competition given the state of competence $\mathbf{q}$. We allow the possibility that the incumbent runs the production facilities at low operation rate after he succeed to exclude the entrant from the market by setting a predatory large 'capacity' q_2. Such low operation rate with a large predatory capacity yields lower profit than he could run the facilities at high (efficient) rate with a smaller capacity. It just means in our model that $q_2 \neq q_2^{BR}(0)$

Financial structure and the cash-in-advance constraint

In our model, there are two types of loans — initial and additional loans. Without an additional loan, the model is seemingly very restrictive. Besides, having two types of loans, we separate a loan to finance precautionary liquidity (the initial loan) from the a loan just to pay the costs (the additional one); one of our propositions (part 1 of Propositions 1 and 2) is the existence of *excess* precautionary liquidity *due to threat of predation*, which is clarified by this separation.

The existence of an additional loan does not however affect the outcome in the product market. We consider various possibilities about additional lending and various types of the initial lender's commitment.

Our "additional loan" includes a wide range of financing instruments, e.g. deferred payment of costs and advance draw of sales. Thus our "cash-in-advance constraint" just means that the entrant should pay out all the capacity cost by his precautionary liquidity, the bank loan (or the additional investment), and such deferred payment of costs and advance draw of sales.

The key in our loan structure is *commitment* of the initial financing on the time of entry. This point leads us to reconsider the meaning of the 'entrant' in our model. We expect an entrant to be subject to the CIA constraint because he is new to the industry and thus has no credit to get deferred payment or advance draw. But, theoretically our 'entrant' can be an incumbent in reality. For example, we should think an entrant as our 'incumbent' if the actual entrant is a large conglomerate and can subside the new business by profits from the other businesses or the entrant is supported unconditionally by the government.

Unverifiability (not unobservability)

Let us consider *unverifiability* of the incumbent's 'capacity' q_2 and the entrant's profit π^1. We should notice the distinction between unverifiability and unobservability. Even if q_2 is unverifiable, the entrant may directly observe q_2 or predict it with high accuracy by good marketing research. He could even present the marketing data about the rival's strategy and its impact on his own business to the lenders so as to convince them of profitability of his entry plan.

What we mean by unverifiability is that anybody (especially the lenders) cannot *legally* verify that such observation and prediction coincide with the actual q_2 (or π^1). Although they could be enough for antitrust lawsuits, here we argue financial lawsuits to enforce the loan repayment. The court needs to know whether the entrant actually has enough money to repay the loan. Furthermore, because the incumbent is a rival in the product market and not legally bound in the entrant's loan contract, it is hard to expect that the incumbent himself would assure to provide verifiable evidence of the actual q_2 for the entrant's lenders, which would help the entrant's financing according to our propositions.

Note that in the unverifiable case, repayment does not eventually rely on the court for enforcement of the contract. The unverifiability prevents the court from enforcing full repayment of the loan. The lender herself has to encourage the entrant to voluntarily repay the whole loan by using liquidation of collateral as threat.

Although we emphasis plausibility of the unverifiable case so far, we do not insist that q_2 is always unverifiable. Our propositions rather suggest an entrant to make things verifiable for better financing. For example, in a Japanese "main bank system" (Hoshi, Kashyap, and Scharfstein, 1991), a borrower has his business activity monitored by "main banks" through keeping all transactions in the bank's account and inviting a banker as an accounting director. This guarantees verifiability of the borrower's liquidity holding and enables the lender to enforce the whole repayment of the loan.

One might feel that our verifiable and unverifiable cases are too extreme. In between, we could think of a stochastically verifiable case, where the lender gets verifiable information about q_2 or π^1 with some probability. On the other hand, so-called "costly state verification", usually meaning that the principal (the lenders) *surely* obtain verifiable information at some cost, should fall into our verifiable case.

Stochastic additional lending

The lender in our model is allowed to commit only to a deterministic policy on an additional loan. This makes discontinuity in the entrant's survival probability to the incumbent's predatory capacity: he stays with probability one or exits with probability one. And, the discontinuity yields the simple non-predation condition as we see in (13).

When the lender took stochastic policy on additional lending, the survival probability could be continuous. Even if the entrant does not have large enough start-up capital to satisfy the non-predation condition $w_0 < \bar{L}^P(q_1^\dagger) - \bar{V}$ and cannot obtain full commitment to keep him stay against the maximal plausible predatory capacity, he might obtain partial commitment. By gradually decreasing the survival probability $P(q_1, q_2^P)$ with the incumbent's capacity size q_2^P, the entrant could keep the incumbent's *expected* predatory profit $P(q_1, q_2^P)\pi^2(q_1^\dagger, q_2^P) + (1 - P(q_1, q_2^P))\pi^2(0, q_2^P)$ below the equilibrium profit $\pi^2(\mathbf{q}^\dagger)$ at any predatory capacity q_2^P. Then the entrant could restrain the incumbent from predation. In a nutshell, the entrant could prevent predation by obtaining appropriate stochastic commitment to credit line, even if he cannot satisfy the non-predation condition (13). (We prove this claim as Theorem 5 in Appendix D, assuming a quasi-direct mechanism.)

But prevention of predation by stochastic commitment is hard to put into practice, compared with deterministic commitment. The entrant and the lender need to reveal to the incumbent *how* the survival probability changes with the incumbent's capacity and that they *commit* to this stochastic schedule of additional lending.

In contrast to stochastic additional lending policy, the amount of precautionary liquidity is easy to observe and to reveal; actually the amount of liquid asset is one of essential accounting information of a company. Sufficient precautionary liquidity simply works to keep the incumbent away from predation. As argued in (9), an initial loan to raise precautionary liquidity is essentially equivalent to a deterministic commitment to credit line.

Equity versus debt financing, and empirical support

In our model, the 'entrant' receives all the remaining profits and asset after the loan repayments, while he unconditionally puts the start-up capital for the business. So the equity investors should be regarded as a party of the 'entrant', not as an initial 'lender' in our model.

Lerner (1995) studies the disk drive industry in 1980–88, seeing the changes in equity financing as shocks to the entrant's financial strength. He tests whether price wars were triggered by entries of financially weak rivals.[17] In 1980–83, a venture company was able to easily raise the start-up capital with equity finance. In this era of "capital market myopia," prices were wholly determined by the products' attributes, independently from the financial weakness of the entrants. In 1984–88 when entrepreneurs suddenly faced difficulty in equity financing, prices were significantly low in the presence of the financially weak rivals.

This empirical result is comparable with Proposition 2 for the unverifiable case. In early 1980s, "capital market myopia" enabled the entrants to raise enough start-up capital w_0. So they sufficed the non-predation condition and could avoid predation. In the late 80s, the difficulty of equity financing forced the entrants to enter the industry with short start-up capital. So they could not obtain sufficient precautionary liquidity and had the incumbents being more aggressive.

[17] Lerner identifies a financially weak firm from two aspects. First, the firm should specialize in disk drive manufacturing, which means the absence of internal financing from other business. Second, the firm's equity capital should be below the median of all the samples.

7 Concluding remarks

We see that threat of predation causes the demand for excess precautionary liquidity that is not spent in equilibrium. Furthermore, we prove that if the incumbent's strategy and thus the entrant's actual profit are unverifiable and the loan contract needs robustness to strategic uncertainty, the entrant faces short supply of excess liquidity and has to shrink his business.

From the discussion we made in detail in Section 6, we can provide policy and practical advices to reduce threat of predation. For example, entrepreneurs and policy makers should allow lenders to monitor the accounting information, provide verifiable evidence about it, and raise enough start-up internal capital from equity market.

We should notice that there must be excess precautionary liquidity even in the verifiable case. First, as we noted in Section 4, the need for excess liquidity would distort the product market outcome if the long-term (initial) loan or the credit line incurs interest or fees; hence threat of predation is a problem in the product market even if the entrant has no difficulty to borrow it. Second, the excess liquidity is just kept to show the entrant's financial healthiness and commitment to stay in the market. It does not contribute to production at all. When liquidity supply is limited in the economy, such demand for precautionary liquidity crowds out real liquidity demand for production and investment (Holmström and Tirole, 1998). Hence the policy that weakens threat of predation improves macroeconomic efficiency through releasing excess liquidity holding.

References

Argenton, C. (2010): "Predation Under Perfect Information," CentER Discussion Paper No. 2010-26, Tilburg University.

Bester, H., and R. Strausz (2001): "Contracting with Imperfect Commitment and the Revelation Principle: the Single Agent Case," *Econometrica*, 69(4), 1077–98.

Bolton, P., and D. S. Scharfstein (1990): "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, 80(1), 93–106.

Elzinga, K. G., and D. E. Mills (2001): "Predatory Pricing and Strategic Theory," *Georgetown Law Journal*, 89(8), 2475–2494.

Fernández-Ruiz, J. (2004): "Predation due to adverse selection in financial markets," *International Journal of Industrial Organization*, 22(5), 715–733.

Fudenberg, D., and J. Tirole (1991): *Game Theory*. MIT Press.

Fumagali, C., and M. Motta (2010): "A Simple Theory of Predation," Centre for Studies in Economics and Finance Working Paper No. 255, University of Naples.

Gerardi, D., and R. Myerson (2007): "Sequential Equilibria in Bayesian Games with Communication," *Games and Economic Behavior*, 60(1), 104–134.

Hart, O. (1995): *Firms, Contracts, and Financial Structure*, Clarendon Lectures in Economics. Oxford University Press.

Holmström, B., and J. Tirole (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106(1), 1–40.

Hoshi, T., A. Kashyap, and D. Scharfstein (1991): "Corporate Structure, Liquidity, and Investment: Evidence from Japanese Industrial Groups," *The Quarterly Journal of Economics*, 106(1), 33–60.

Khanna, N., and M. D. Schroder (2010): "Optimal Debt Contracts and Product Market Competition with Exit and Entry," *Journal of Economic Theory*, 145(1), 156–188.

Lerner, J. (1995): "Pricing and Financial Resources: an Analysis of the Disk Drive Industry, 1980–88," *Review of Economics and Statistics*, 77(4), 585–598.

Méndez-Naya, L., I. García-Jurabo, and J. Cesco (1995): "Perfection of Nash equilibria in continuous games," *Mathematical Social Sciences*, 29(3), 225–237.

Osborne, M. J., and A. Rubinstein (1994): *A Course in Game Theory*. MIT Press.

Poitevin, M. (1989): "Financial Signaling and the "Deep-pocket" Argument," *RAND Journal of Economics*, 20(1), 26–40.

Roth, D. (1996): "Rationalizable Predatory Pricing," *Journal of Economic Theory*, 68(2), 380–396.

Snyder, C. M. (1996): "Negotiation and Renegotiation of Optimal Financing Contracts under the Threat of Predation," *Journal of Industrial Economics*, 44(3), 325–343.

TAYLOR, D., AND J. S. ARCHER (1994): *Up Against the Walmarts: how your business can prosper in the shadow of the retail giants*. American Management Association.

TELSER, L. G. (1966): "Cutthroat Competition and the Long Purse," *Journal of Law and Economics*, 9, 259–277.

Online appendix for "Excess Liquidity against Predation"

Dai Zusai

A Proof of the Revelation Principle (Theorem 3)

In this appendix, we consider the case where q_2 is unverifiable and the additional loan is not committed and show the revelation principle on sequential equilibrium in the game after the contract is accepted (**the post-entry game**). We can easily extend the results (Lemma 1 and Theorem 3) to the cases where an additional loan is prohibited or committed.

We first define strategy space in the post-entry game and characterize sequential equilibrium. Then, we obtain the revelation principle for these sequential equilibria.

Finite strategy space

We assume that the numbers of feasible capacity sizes and available messages are (arbitrarily) finite and at least two.[18]

Under the contract $\mathcal{C}$ with the message space M, the space of mixed (behavioral) strategies is given as follows.[19]

$$\begin{aligned} \text{The entrant's capacity:} \quad & \sigma_1 \in \Delta Q_1. \\ \text{The incumbent's capacity:} \quad & \sigma_2 \in \Delta Q_2. \\ \text{The entrant's message given } (q_1, q_2) \in Q_1 \times Q_2\text{:} \quad & \sigma_m(\cdot|q_1, q_2) \in \Delta M. \\ \text{The additional lender's decision given } (m, q_1) \in M \times Q_1 \quad & \sigma_a(\cdot|q_1, m) \in \Delta A. \end{aligned}$$

Here the set A consists of 0 (rejecting the loan) and 1 (accepting the loan). Let Σ be the space of feasible strategy profiles $\sigma = (\sigma_1, \sigma_2, \sigma_m, \sigma_a)$. The posterior belief $\mu(\cdot|q_1, m) \in \Delta Q_2$ is a probability measure on Q_2, conditional on $q_1 \in Q_1$ and $m \in M$.

Sequential equilibrium

Here we characterize sequential equilibrium (σ^*, μ^*) in the post-entry game.

[18] The capacity space Q_i is bounded. It is justified if there exists $\bar{q}_i < \infty$ such that $\pi^i(\bar{q}_i, 0) = 0$ and $\partial\pi^i/\partial q_i(\bar{q}_i, 0) < 0$; since a larger capacity than $\bar{q}_i$ results loss (negative profit) no matter to the rival's capacity, the firm i would not choose such a huge capacity size.

[19] ΔX denotes the set of probability measures on the set X. In particular, with a finite set X, ΔX is an $\sharp X$-dimensional simplex, i.e., $\Delta X := \{\sigma \in \mathbf{R}_+^{\sharp X} \mid \sum_{x \in X} \sigma(x) = 1\}$.

The posterior belief μ^* must be consistent: there exists a sequence of completely mixed strategies $\{\sigma^k\} \subset \mathring{\Sigma}$ that converges to σ^* and for each $q_2 \in Q_2, q_1 \in Q_1, m \in M$

$$\mu^k(q_2|q_1,m) := \frac{\sigma_m^k(m|q_1,q_2)\sigma_2^k(q_2)}{\sum_{q_2' \in Q_2} \sigma_m^k(m|q_1,q_2')\sigma_2^k(q_2')} \longrightarrow \mu^*(q_2|q_1,m)$$

as $k \to \infty$.[20]

When the additional lending is not committed, the additional lender optimizes the lending strategy $\sigma_a(\cdot|q_1,m) \in \Delta A$ so as to maximize

$$\begin{aligned} & \mathbb{E}_{a,q_2}\left[a(\min\{0,\pi^1(q_1,q_2)+B\})|q_1,m\right] \\ = \quad & \sigma_a(1|q_1,m)\sum_{q_2 \in Q_2}(\min\{0,\pi^1(q_1,q_2)+B\})\mu^*(q_2|q_1,m), \end{aligned}$$

given $(q_1,m) \in Q_1 \times M$. Hence the optimal strategy of the additional lender is $\sigma_a^*(1|q_1,m) = 0$, if $\pi^1(q_1,q_2)+B < 0$ for some q_2 in the support of $\mu^*(q_2|q_1,m)$. Otherwise, $a=0$ and $a=1$ are indifferent. We focus on equilibrium where the additional lending policy is either $a=1$ or $a=0$ deterministically in the equilibrium: $\sigma_a^*(1|q_1,m) \in \{0,1\}$.[21]

The entrant optimizes the messaging strategy $\sigma_m(\cdot|q_1,q_2) \in \Delta M$ so as to maximize

$$\begin{aligned} & \mathbb{E}_{a,m}\left[a\left\{\max\{0,\pi^1(q_1,q_2)+B\}+\bar{V}-\delta(m|q_1)\right\}+(1-a)(\bar{V}+w_0)|q_1,q_2\right] \\ & \quad = \bar{V}+w_0+\sum_{m \in M}\sigma_a^*(1|q_1,m)\left[\max\{0,\pi^1(q_1,q_2)+B\}-\delta(m|q_1)-w_0\right]\sigma_m(m|q_1,q_2), \end{aligned}$$

given $(q_1,q_2) \in Q_1 \times Q_2$. Let $\underline{\delta}(q_1)$ be the minimal total repayment when the entrant stays in the market:

$$\underline{\delta}(q_1) := \min\{\delta(m|q_1)|\sigma_a^*(1|q_1,m)=1\}.$$

In his optimal messaging strategy, a message m with $\sigma_a^*(1|q_1,m)=1$ at each (q_1,m) cannot be sent if $\delta(m|q_1) > \underline{\delta}(q_1)$.

Given that the equilibrium additional lending policy is either $\sigma_a^*=0$ or $\sigma_a^*=1$, we can

[20] $\mathring{X}$ is the interior of a set X.

[21] If the additional lending is prohibited, $\sigma_a^*(1|\cdot) \equiv 0$. If the credit line is provided, the policy σ_a is committed in the contract $\mathcal{C}$.

classify all the sent messages into two sets:

$$M_1(q_1) := \{m \in M | \sigma_a^*(1|q_1, m) = 1, \exists q_2 \in Q_2 \ \sigma_m^*(m|q_1, q_2) > 0\}$$
$$M_0(q_1) := \{m \in M | \sigma_a^*(1|q_1, m) = 0, \exists q_2 \in Q_2 \ \sigma_m^*(m|q_1, q_2) > 0\}$$

Here $M_1(q_1)$ ($M_0(q_1)$, resp) is the set of messages that satisfies $\sigma_a^*(1|q_1, m) = 1$ ($\sigma_a^*(1|q_1, m) = 0$, resp) and is actually sent with positive probability at some q_2. Given (q_1, q_2), let $P^*(q_1, q_2)$ be the equilibrium probability of staying in the market, i.e., the probability of sending message in $M_1(q_1)$:

$$P^*(q_1, q_2) := \sum_{m \in M} \sigma_m^*(m'|q_1, q_2)\sigma_a^*(1|q_1, m) = \sum_{m' \in M_1(q_1)} \sigma_m^*(m'|q_1, q_2).$$

Given q_1, $Q_2^S(q_1)$ is the set of capacity sizes q_2 at which the entrant stays in the market with positive probability in the original equilibrium:

$$Q_2^S(q_1) := \{q_2 \in Q_2 | P^*(q_1, q_2) > 0\}.$$

Suppose that both $M_0(q_1)$ and $M_1(q_1)$ are nonempty at $q_1 \in Q_1$. The entrant sends only messages in the set $M_1(q_1)$ ($M_0(q_1)$, resp.) with positive probability and $P^*(q_1, q_2)$ is one (zero, resp.), if $\max\{0, \pi^1(q_1, q_2) + B\} - \underline{\delta}(q_1) - w_0$ is positive (negative, resp). If it is just zero, messages in both sets can be sent. As long as $w_0 > 0$, a message $m \in M_1(q_1)$ is sent after the capacities are set to $\mathbf{q}$, only if $\pi^1(\mathbf{q}) + B > 0$; $\sigma_m^*(m|\mathbf{q}) > 0$ requires $\max\{0, \pi^1(q_1, q_2) + B\} \geq \underline{\delta}(q_1) + w_0 > 0$ by $\underline{\delta}(q_1) \geq 0$. Hence any $q_2 \in Q_2^S(q_1)$ satisfies $\pi^1(q_1, q_2) + B > 0$.

The incumbent optimizes the capacity strategy $\sigma_2 \in \Delta Q_2$ so as to maximize

$$\begin{aligned}
&\mathbb{E}_{a,m,q_1,q_2}\left[a\pi^2(q_1, q_2) + (1 - a)\pi^2(0, q_2)\right] \\
= &\sum_{q_1 \in Q_1} \sum_{q_2 \in Q_2} \sum_{m \in M} \left\{\sigma_a^*(1|q_1, m)\pi^2(q_1, q_2) + \sigma_a^*(0|q_1, m)\pi^2(0, q_2)\right\} \sigma_m^*(m|q_1, q_2)\sigma_2(q_2)\sigma_1^*(q_1),
\end{aligned}$$

From the above argument, this reduces to

$$\sum_{q_1\in Q_1}\Big[\sum_{q_2\in Q_2^S(q_1)}\left\{P^*(q_1,q_2)\pi^2(q_1,q_2)+(1-P^*(q_1,q_2))\pi^2(0,q_2)\right\}\sigma_2(q_2)$$
$$+\sum_{q_2\notin Q_2^S(q_1)}\pi^2(0,q_2)\sigma_2(q_2)\Big]\sigma_1^*(q_1). \quad (17)$$

The entrant optimizes the capacity strategy $\sigma_1\in\Delta Q_1$ so as to maximize

$$\bar V+w_0+\mathbb{E}_{a,m,q_1,q_2}\left[a(\max\{0,\pi^1(q_1,q_2)+B\}-\delta(m|q_1)-w_0)\right]$$

From the above argument, this reduces to

$$\bar V+w_0+\sum_{q_1\in Q_1}\sum_{q_2\in Q_2^S(q_1)}\left[\pi^1(q_1,q_2)-\underline{\delta}(q_1)+B-w_0\right]\sigma_2^*(q_2)\sigma_1(q_1). \quad (18)$$

Lemma 1. *Suppose $w_0>0$. Consider a sequential equilibrium*[22] *of the post-entry game in the case where q_2 is unverifiable and the additional loan is not committed.*
(a) If the additional lending is not committed, the additional lender's equilibrium strategy $\sigma_a^*(\cdot|q_1,m)$ is

$$\begin{aligned}\sigma_a^*(1|q_1,m)&\in[0,1] && \text{if } \pi^1(q_1,q_2)+B\geq 0 \quad \text{for all } q_2\in \text{support}(\mu^*(q_2|q_1,m)),\\ &=0 && \text{if } \pi^1(q_1,q_2)+B<0 \quad \text{for some } q_2\in \text{support}(\mu^*(q_2|q_1,m)).\end{aligned}$$

Furthermore, focus on an equilibrium where the additional lender employs a pure strategy, i.e., $\sigma_a^*(1|q_1,m)$ is either 0 or 1.
(b) i) Given q_1, suppose that both $\{m|\sigma_a^*(1|q_1,m)=1\}$ and $\{m|\sigma_a^*(1|q_1,m)=0\}$ are nonempty sets (a separating case). Then, $M_0(q_1)$ and $M_1(q_1)$ are nonempty. Any $m\in M_1(q_1)$ satisfies $\delta(m|q_1)=\underline{\delta}(q_1)$. Any $q_2\in Q_2^S(q_1)$ satisfies $\pi^1(q_1,q_2)+B>0$. The optimal strategy $\hat\sigma_m^*$ satisfies

$$\begin{cases}[\hat\sigma_m^*(m|\mathbf{q})>0\Rightarrow m\in M_1(q_1)] \text{ and thus } P^*(\mathbf{q})=1 & \text{if } \pi^1(\mathbf{q})+B>\underline{\delta}(q_1)+w_0,\\ [\hat\sigma_m^*(m|\mathbf{q})>0\Rightarrow m\in M_0(q_1)] \text{ and thus } P^*(\mathbf{q})=0 & \text{if } \pi^1(\mathbf{q})+B<\underline{\delta}(q_1)+w_0,\\ [\hat\sigma_m^*(m|\mathbf{q})>0\Rightarrow m\in M_1(q_1)\cup M_0(q_1)] \text{ and thus } P^*(\mathbf{q})\in[0,1] & \text{otherwise,}\end{cases}$$

[22] So far we do not rely on consistency of the belief to characterize the optimal strategies. These properties should hold without it, i.e., in any (weak) perfect Bayesian equilibria, not only in sequential equilibria.

ii) If $\{m|\sigma_a^*(1|q_1,m)=0\}=\emptyset$ (a pooling-stay case), then $M_0(q_1)=\emptyset$ and $P^*(q_1,q_2)=1$ for any $q_2\in Q_2$. For any $m\in M_1(q_1)$, we have $\delta(m|q_1)=\underline{\delta}(q_1)$ and any q_2 in the support of $\mu^*(q_2|q_1,m)$ satisfies $\pi^1(q_1,q_2)+B\geq 0$.

iii) If $\{m|\sigma_a^*(1|q_1,m)=0\}=\emptyset$(a pooling-exit case), then $M_1(q_1)=\emptyset$ and $P^*(q_1,q_2)=0$ for any $q_2\in Q_2$. For any $m\in M_0(q_1)$, there exists some q_2 in the support of $\mu^*(q_2|q_1,m)$ such that $\pi^1(q_1,q_2)+B<0$.

(c) The incumbent's equilibrium capacity strategy σ_2^* maximizes (17), given σ_1^* and $P^*(q_1,q_2)$. As well, the entrant's equilibrium capacity strategy σ_1^* maximizes (18), given σ_2^*.

Revelation principle

So far we formalize the finite subgame after period 1 and its sequential equilibrium, under an arbitrary mechanism $\mathcal{C}$ with an arbitrary message space M. Here we obtain the revelation principle: a sequential equilibria under a contract $\mathcal{C}$ with a message space M is reduced to a sequential equilibrium in the *quasi*-direct mechanism $\hat{\mathcal{C}}$ with the message space $\hat{M}(q_1)=M_0(q_1)\cup Q_2^S(q_1)$ for each q_1.

In the quasi-direct mechanism, the entrant announces the true q_2 if he wants to stay in the market; otherwise he sends any message in $M_0(q_1)$ so that the additional lender rejects the loan. We keep all the messages of $M_0(q_1)$ in our message space, so as to retain the belief from these messages that induce the exit.

Theorem 3. *Consider the case where q_2 is unverifiable and the additional loan is not committed. Assume $w_0>0$. Suppose that a mixed (behavioral) strategy profile $\sigma^*=\{\sigma_1^*,\sigma_2^*,\sigma_m^*,\sigma_a^*\}$ is a sequential equilibrium where the additional lender employs a pure strategy $\sigma_a^*(1|q_1,m)\in\{0,1\}$, under a mechanism $\mathcal{C}=\{B-w_0,M,D,\beta\}$ with a message space M, $D:M\otimes Q_1\to\mathbb{R}_+$ and $\beta:M\otimes Q_1\to[0,1]$.*

Then, there exists a sequential equilibrium $(\hat{\sigma}^*,\hat{\mu}^*)$ that results in the same capacity strategies (σ_1^*,σ_2^*) and the same probability $P^*:Q\to[0,1]$ that the entrant stays in the market, under a quasi-direct mechanism $\hat{\mathcal{C}}=\{B-w_0,\hat{M},\hat{D},\hat{\beta}\}$ with the message space $\hat{M}(q_1)=M_0(q_1)\cup Q_2^S(q_1)$, $\hat{D}:\hat{M}\otimes Q_1\to\mathbb{R}_+$, and $\hat{\beta}(\cdot|q_1):\hat{M}\otimes Q_1\to[0,1]$.

Here, $\hat{\mathcal{C}}$ satisfies

$$\hat{\delta}(\tilde{q}_2|q_1):=\hat{D}(\tilde{q}_2|q_1)+\hat{\beta}(\tilde{q}_2|q_1)\bar{V}=\underline{\delta}(q_1)\quad \textit{for each } q_1\in Q_1,\tilde{q}_2\in Q_2^S(q_1), \tag{19}$$

as well as $\hat{D}(m|q_1):=D(m|q_1),\hat{\beta}(m|q_1):=\beta(m|q_1)$ for each $m\in M_0(q_1)$.

The profile $(\hat{\sigma}^*, \hat{\mu}^*)$ is specified as follows:

$$(\hat{\sigma}^*) \qquad \hat{\sigma}_i^*(q_i) := \sigma_i^*(q_i) \quad \textit{for each } q_i \in Q_i, i \in \{1,2\};$$

(Separating case) If both $M_0(q_1)$ and $M_1(q_1)$ are nonempty, then

$$(\hat{\sigma}^*) \begin{cases} \hat{\sigma}_m^*(\tilde{q}_2|q_1,q_2) := 0 & \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1) \setminus \{q_2\}, \\ \hat{\sigma}_m^*(q_2|q_1,q_2) := P^*(q_1,q_2) & \textit{for each } q_2 \in Q_2^S(q_1), \\ \hat{\sigma}_m^*(m|q_1,q_2) := \sigma_m^*(m|q_1,q_2) & \textit{for each } q_1 \in Q_1, q_2 \in Q_2, m \in M_0(q_1), \\ \hat{\sigma}_a^*(1|q_1,\tilde{q}_2) := 1 & \textit{for each } \tilde{q}_2 \in Q_2^S(q_1), \\ \hat{\sigma}_a^*(1|q_1,m) := 0 & \textit{for each } m \in M_0(q_1); \end{cases}$$

$$(\hat{\mu}^*) \begin{cases} \hat{\mu}^*(q_2|q_1,\tilde{q}_2) := I(q_2,\tilde{q}_2) & \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1), \\ \hat{\mu}^*(q_2|q_1,m) := \mu^*(q_2|q_1,m) & \textit{for each } q_2 \in Q_2, m \in M_0(q_1). \end{cases}$$

Here $I(q_2,\tilde{q}_2)$ is the indicator function for $q_2 = \tilde{q}_2$.

(Pooling-stay case) If $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$, then $\hat{M}(q_1) = Q_2^S(q_1) = Q_2$ and

$$(\hat{\sigma}^*) \begin{cases} \hat{\sigma}_m^*(\tilde{q}_2|q_1,q_2) := 1/\sharp Q_2 & \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2, \\ \hat{\sigma}_a^*(1|q_1,\tilde{q}_2) := 1 & \textit{for each } \tilde{q}_2 \in Q_2^S(q_1), \end{cases}$$

$$(\hat{\mu}^*) \quad \hat{\mu}^*(q_2|q_1,\tilde{q}_2) := \sigma_2^*(q_2), \quad \textit{for each } q_2 \in Q_2, \tilde{q}_2 \in Q_2.$$

(Pooling-exit case) If $M_0(q_1) \neq \emptyset$ and $M_1(q_1) = \emptyset$, then $\hat{M}(q_1) = M_0(q_1)$ and

$$(\hat{\sigma}^*) \begin{cases} \hat{\sigma}_m^*(m|q_1,q_2) := \sigma_m^*(m|q_1,q_2) & \textit{for each } q_2 \in Q_2, m \in M_0(q_1), \\ \hat{\sigma}_a^*(1|q_1,m) := 0 & \textit{for each } m \in M_0(q_1); \end{cases}$$

$$(\hat{\mu}^*) \quad \hat{\mu}^*(q_2|q_1,m) := \mu^*(q_2|q_1,m) \quad \textit{for each } q_2 \in Q_2, m \in M_0(q_1).$$

Proof. In this proof, we consider the uncommitted additional lending; the proof for the committed lending (credit line or no additional loan) is obtained just by eliminating the part that regards the additional lender's strategy from this proof.

Define the repayment $\hat{D}$ and the liquidation policy $\hat{\beta}$ for each $q_1 \in Q_1$ and $\tilde{q}_2 \in Q_2^S(q_1)$

as

$$\hat{D}(\tilde{q}_2|q_1) := \sum_{m \in M_1(q_1)} D(m|q_1)p^*(m|q_1, \tilde{q}_2),\ \hat{\beta}(\tilde{q}_2|q_1) := \sum_{m \in M_1(q_1)} \beta(m|q_1)p^*(m|q_1, \tilde{q}_2).$$

Here $p^*(m|q_1, q_2)$ is the probability to send the message $m \in M_1(q_1)$ in the original equilibrium given capacities (q_1, q_2), conditional on stay in the market:[23]

$$p^*(m|q_1, q_2) := \sigma_m^*(m|q_1, q_2)/P^*(q_1, q_2).$$

Then, Lemma 1 implies (19).

We show that the strategy profile $\hat{\sigma}^* = \{\sigma_1^*, \sigma_2^*, \hat{\sigma}_m^*, \hat{\sigma}_a^*\}$ specified in the theorem is a sequential equilibrium under the belief $\hat{\mu}^*$.

Consistency of belief. From the sequence of completely mixed strategy profiles $\{\sigma^k\}$ converging to σ^* in the original sequential equilibrium, we define the sequence $\{\hat{\sigma}^k\}$ and then prove the consistency of the belief $\hat{\mu}^*$.

For each $k \in \mathbf{N}$, define $\{\hat{\sigma}_1^k, \hat{\sigma}_2^k\}$ as

$$\begin{aligned} \hat{\sigma}_1^k(q_1) &:= \sigma_1^k(q_1) && \in (0,1), \\ \hat{\sigma}_2^k(q_2) &:= \frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2) && \in (0,1). \end{aligned}$$

Since $\sigma_i^k \to \sigma_i^*$, we have $\hat{\sigma}_i^k \to \sigma_i^* = \hat{\sigma}_i^*$ for each $i \in \{1, 2\}$.

Separating case Consider $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. For each $k \in \mathbf{N}$, define $\{\hat{\sigma}_m^k, \hat{\sigma}_a^k\}$ as

$$\begin{aligned} \hat{\sigma}_m^k(\tilde{q}_2|q_1, q_2) &:= \frac{1}{k\#Q_2^S(q_1)} + \left(1 - \frac{1}{k}\right) I(q_2, \tilde{q}_2)P^k(q_1, q_2) && \in (0,1), \\ \hat{\sigma}_m^k(m|q_1, q_2) &:= \left(1 - \frac{1}{k}\right)\left\{\sigma_m^k(m|q_1, q_2) + \frac{\left(1 - I^S(q_2|q_1)\right)P^k(q_1, q_2)}{\#M_0(q_1)}\right\} && \in (0,1), \\ \hat{\sigma}_a^k(1|q_1, \tilde{q}_2) &:= 1 - 1/k && \in (0,1), \\ \hat{\sigma}_a^k(1|q_1, m) &:= 1/k && \in (0,1) \end{aligned}$$

[23] Here P^* is based on the original equilibrium σ_m^* and $P^*(q_1, q_2)$ is positive iff $q_2 \in Q_2^S(q_1)$.

for each $q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1), m \in M_0(q_1)$. Here $P^k : Q \to (0,1)$ is given by

$$P^k(q_1,q_2) := \sum_{m' \in M/M_0(q_1)} \sigma_m^k(m'|q_1,q_2) \ \in (0,1),$$

and $I^S(q_2|q_1)$ is the indicator for $q_2 \in Q_2^S(q_1)$. Notice that $\sum_{\tilde{q}_2 \in Q_2^S(q_1)} I(q_2,\tilde{q}_2) = I^S(q_2|q_1)$ and $P^k(q_1,q_2) + \sum_{m \in M_0(q_1)} \sigma_m^k(m|q_1,q_2) = 1$. Here $\hat{\sigma}_m^k(\cdot|q_1,q_2)$ belongs to the interior of $\Delta\hat{M}(q_1)$ for all $\mathbf{q} \in Q$, since

$$\begin{aligned}
&\sum_{\tilde{q}_2 \in Q_2^S(q_1)} \hat{\sigma}_m^k(\tilde{q}_2|\mathbf{q}) + \sum_{m' \in M_0(q_1)} \hat{\sigma}_m^k(m'|\mathbf{q}) \\
=&\frac{1}{k} + \left(1 - \frac{1}{k}\right)\left\{\left(\sum_{\tilde{q}_2 \in Q_2^S(q_1)} I(q_2,\tilde{q}_2) + 1 - I^S(q_2|q_1)\right) P^k(\mathbf{q}) + \sum_{m \in M_0(q_1)} \sigma_m^k(m|\mathbf{q})\right\} = 1,
\end{aligned}$$

As specified in ($\hat{\sigma}^*$), $\hat{\sigma}_a^*$ is the limit of $\hat{\sigma}_a^k$ as $k \to \infty$. According to Lemma 1 (b), we obtain $P^k \to P^*$ and thus $\hat{\sigma}_m^k \to \hat{\sigma}_m^*$ as $k \to \infty$.

The Bayesian belief $\hat{\mu}^k$ determined from $(\hat{\sigma}_2^k, \hat{\sigma}_m^k)$ actually converges to $\hat{\mu}^*$. For a while, omit q_1 from arguments in functions. For each $q_2 \in Q_2, m \in M_0$, the belief is

$$\begin{aligned}
&\hat{\mu}^k(q_2|m) \\
:=& \frac{\hat{\sigma}_m^k(m|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_m^k(m|q_2')\hat{\sigma}_2^k(q_2')} \\
=& \frac{\left(1-\frac{1}{k}\right)\left\{\sigma_m^k(m|q_2) + \frac{(1-I^S(q_2))P^k(q_2)}{\#M_0}\right\}\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2)\right\}}{\sum_{q_2' \in Q_2}\left(1-\frac{1}{k}\right)\left\{\sigma_m^k(m|q_2') + \frac{(1-I^S(q_2'))P^k(q_2')}{\#M_0}\right\}\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2')\right\}} \\
=& \frac{\frac{1}{\sqrt{k}\#Q_2}\left\{\sigma_m^k(m|q_2) + \frac{(1-I^S(q_2))P^k(q_2)}{\#M_0}\right\} + \left(1-\frac{1}{\sqrt{k}}\right)\left\{\mu^k(q_2|m)S^k(m) + \frac{\sigma_2^k(q_2)(1-I^S(q_2))P^k(q_2)}{\#M_0}\right\}}{\frac{1}{\sqrt{k}\#Q_2}\left\{s^k(m) + \sum_{q_2' \notin Q_2^S}\frac{P^k(q_2')}{\#M_0}\right\} + \left(1-\frac{1}{\sqrt{k}}\right)\left\{S^k(m) + \sum_{q_2' \notin Q_2^S}\frac{\sigma_2^k(q_2')P^k(q_2')}{\#M_0}\right\}},
\end{aligned}$$

where $s^k(m) := \sum_{q_2' \in Q_2} \sigma_m^k(m|q_2')$ converges to $s^*(m) := \sum_{q_2' \in Q_2} \sigma_m^*(m|q_2') \in [0,\infty)$ and $S^k(m) := \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_2')$ to $S^*(m) := \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_2') \in [0,\infty)$, and $\mu^k(q_2|m) = \sigma_2^k(q_2)\sigma_m^k(m|q_2)/S^k(m) \to \mu^*(q_2|m) \in [0,1]$. By construction, $(1-I^S(q_2))P^*(q_2) = 0$ for any q_2, since $I^S(q_2) = 0$ for any $q_2 \in Q_2^S$ and $P^*(q_2) = 0$ for any $q_2 \notin Q_2^S$. Hence we have

$$\begin{aligned}
\lim_{k\to\infty} \hat{\mu}^k(q_2|m) &= \frac{0 \times \left\{\sigma_m^*(m|q_2) + \frac{0}{\#M_0}\right\} + 1 \times \left\{\mu^*(q_2|m)S^*(m) + \frac{\sigma_2^*(q_2)\cdot 0}{\#M_0}\right\}}{0 \times \left\{s^*(m) + \sum_{q_2' \notin Q_2^S}\frac{0}{\#M_0}\right\} + 1 \times \left\{S^*(m) + \sum_{q_2' \notin Q_2^S}\frac{\sigma_2^*(q_2')\cdot 0}{\#M_0}\right\}} \\
&= \mu^*(q_2|m).
\end{aligned}$$

For $\tilde{q}_2 \in Q_2^S(q_1), q_2 \in Q_2/\{\tilde{q}_2\}$, the belief is

$$
\begin{aligned}
\hat{\mu}^k(q_2|\tilde{q}_2) &:= \frac{\hat{\sigma}_m^k(\tilde{q}_2|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_m^k(\tilde{q}_2|q_2')\hat{\sigma}_2^k(q_2')} = \frac{\frac{1}{k\#Q_2^S}\hat{\sigma}_2^k(q_2)}{\frac{1}{k\#Q_2^S} + \left(1-\frac{1}{k}\right)P^k(\tilde{q}_2)\hat{\sigma}_2^k(\tilde{q}_2)} \\
&= \left[\frac{1}{\hat{\sigma}_2^k(q_2)} + (k-1)\#Q_2^S \frac{\hat{\sigma}_2^k(\tilde{q}_2)}{\hat{\sigma}_2^k(q_2)} P^k(\tilde{q}_2)\right]^{-1} \\
&< \left[(k-1)\#Q_2^S \frac{\hat{\sigma}_2^k(\tilde{q}_2)}{\hat{\sigma}_2^k(q_2)} P^k(\tilde{q}_2)\right]^{-1} \qquad (\because \hat{\sigma}_2^k(q_2) > 0) \\
&= \left[(k-1)\#Q_2^S \frac{1+\left(\sqrt{k}-1\right)\#Q_2\sigma_2^k(\tilde{q}_2)}{1+\left(\sqrt{k}-1\right)\#Q_2\sigma_2^k(q_2)} P^k(\tilde{q}_2)\right]^{-1} \\
&< \left[(k-1)\#Q_2^S \frac{1}{1+\left(\sqrt{k}-1\right)\#Q_2} P^k(\tilde{q}_2)\right]^{-1} \qquad (\because \sigma_2^k(\tilde{q}_2) > 0, \sigma_2^k(q_2) < 1) \\
&= \left(\frac{1}{(k-1)\#Q_2^S} + \frac{\#Q_2}{(\sqrt{k}+1)\#Q_2^S}\right)\frac{1}{P^k(\tilde{q}_2)}.
\end{aligned}
$$

As $\hat{\mu}^k(q_2|\tilde{q}_2) > 0$ and $P^*(\tilde{q}_2) > 0$ for any $\tilde{q}_2 \in Q_2^S$, this implies

$$
\begin{aligned}
&0 \le \lim_{k\to\infty} \hat{\mu}^k(q_2|\tilde{q}_2) \le 0/P^*(\tilde{q}_2) = 0, \\
&\therefore \quad \lim_{k\to\infty} \hat{\mu}^k(q_2|\tilde{q}_2) = 0 = \hat{\mu}^*(q_2|\tilde{q}_2).
\end{aligned}
$$

Because this holds for all $q_2 \in Q_2/\{\tilde{q}_2\}$, we have

$$
\lim_{k\to\infty} \hat{\mu}^k(\tilde{q}_2|\tilde{q}_2) = 1 = \hat{\mu}^*(\tilde{q}_2|\tilde{q}_2).
$$

Therefore the belief $\hat{\mu}^*$ specified in ($\hat{\mu}^*$) is actually consistent with $\hat{\sigma}^*$.

Pooling-stay case Consider $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. For each $k \in \mathbf{N}$, define $\{\hat{\sigma}_m^k, \hat{\sigma}_a^k\}$ as

$$
\begin{aligned}
\hat{\sigma}_m^k(\tilde{q}_2|q_1, q_2) &:= 1/(k\#Q_2) \quad \in (0,1), \\
\hat{\sigma}_a^k(1|q_1, \tilde{q}_2) &:= 1 - 1/k \quad \in (0,1),
\end{aligned}
$$

for each $q_2, \tilde{q}_2 \in Q_2$. Then the strategies are completely mixed and converge to σ^* given such q_1.

For a while, we omit q_1 from arguments of functions. For $\tilde{q}_2, q_2 \in Q_2$, the Bayesian belief is determined as

$$\hat{\mu}^k(q_2|\tilde{q}_2) = \frac{\hat{\sigma}_m^k(\tilde{q}_2|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_m^k(\tilde{q}_2|q_2')\hat{\sigma}_2^k(q_2')} = \frac{\hat{\sigma}_2^k(q_2)/(k\#Q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_2^k(q_2')/(k\#Q_2)} = \hat{\sigma}_2^k(q_2).$$

Hence we have

$$\lim_{k\to\infty} \hat{\mu}^k(\tilde{q}_2|q_2) = \hat{\sigma}_2^*(q_2).$$

Therefore the belief $\hat{\mu}^*$ specified in $(\hat{\mu}^*)$ is actually consistent with $\hat{\sigma}^*$.

Pooling-exit case Consider $q_1 \in Q_1$ such that $M_0(q_1) \neq \emptyset$ and $M_1(q_1) = \emptyset$. For each $k \in \mathbf{N}$, define $\{\hat{\sigma}_m^k, \hat{\sigma}_a^k\}$ as

$$\begin{aligned} \hat{\sigma}_m^k(m|q_1, q_2) &:= \sigma_m^k(m|q_1, q_2) \quad \in (0,1); \\ \hat{\sigma}_a^k(1|q_1, m) &:= 1/k \quad \in (0,1) \end{aligned}$$

for each $q_2 \in Q_2, m \in M_0(q_1)$.

For a while, we omit q_1 from arguments of functions. For each $q_2 \in Q_2, m \in M_0$, the Bayesian belief is determined as

$$\begin{aligned} \hat{\mu}^k(q_2|m) &:= \frac{\hat{\sigma}_m^k(m|q_2)\hat{\sigma}_2^k(q_2)}{\sum_{q_2' \in Q_2} \hat{\sigma}_m^k(m|q_2')\hat{\sigma}_2^k(q_2')} = \frac{\sigma_m^k(m|q_2)\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2)\right\}}{\sum_{q_2' \in Q_2} \sigma_m^k(m|q_2')\left\{\frac{1}{\sqrt{k}\#Q_2} + \left(1 - \frac{1}{\sqrt{k}}\right)\sigma_2^k(q_2')\right\}} \\ &= \frac{\frac{1}{\sqrt{k}\#Q_2}\sigma_m^k(m|q_2) + \left(1 - \frac{1}{\sqrt{k}}\right)\mu^k(q_2|m)S^k(m)}{\frac{1}{\sqrt{k}\#Q_2}s^k(m) + \left(1 - \frac{1}{\sqrt{k}}\right)S^k(m)}, \end{aligned}$$

where $s^k(m) := \sum_{q_2' \in Q_2} \sigma_m^k(m|q_2')$ converges to $s^*(m) := \sum_{q_2' \in Q_2} \sigma_m^*(m|q_2') \in [0,\infty)$, and $S^k(m) := \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_2')$ to $S^*(m) := \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_2') \in [0,\infty)$, and $\mu^k(q_2|m) = \sigma_2^k(q_2)\sigma_m^k(m|q_2)/S^k(m) \to \mu^*(q_2|m) \in [0,1]$. Hence we have

$$\lim_{k\to\infty} \hat{\mu}^k(q_2|m) = \frac{0 \times \sigma_m^*(m|q_2) + 1 \times \mu^*(q_2|m)S^*(m)}{0 \times s^*(m) + 1 \times S^*(m)} = \mu^*(q_2|m).$$

Therefore the belief $\hat{\mu}^*$ specified in $(\hat{\mu}^*)$ is actually consistent with $\hat{\sigma}^*$.

Sequential rationality. We prove the optimality of the strategy profile $\hat{\sigma}^*$ given the belief $\hat{\mu}^*$. Given $(\hat{\sigma}_a^*, \hat{\sigma}_m^*)$, the probability for the entrant to stay is the same probability as $P^*(q_1, q_2)$ in the original equilibrium. Hence the incumbent's expected profit under any strategy σ_2 remains the same, given the entrant's capacity strategy σ_1^*. So does the entrant's. Therefore $(\hat{\sigma}_1^*, \hat{\sigma}_2^*) = (\sigma_1^*, \sigma_2^*)$ is still the optimal capacity strategy in the equilibrium $(\hat{\sigma}^*, \hat{\mu}^*)$. Next, we check the optimality of the message strategy and of the additional lending strategy in each case.

Separating case Consider $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. According to Lemma 1 (a), the additional lender's strategy $\hat{\sigma}_a^*$ specified in (σ^*) is optimal under the belief $\hat{\mu}^*$. In particular, $\mu^*(q_2|q_1, \tilde{q}_2)$ is positive only at $q_2 = \tilde{q}_2$ and $\tilde{q}_2 \in Q_2^S(q_1)$ implies $\pi^1(q_1, \tilde{q}_2) + B > 0$ by Lemma 1 (b); thus $\hat{\sigma}_a^*(1|q_1, \tilde{q}_2) = 1$ is optimal for any message $\tilde{q}_2 \in Q_2^S(q_1)$. Note that $\hat{\sigma}_a^*(1|q_1, m) = 0$ is always optimal.

Applying Lemma 1 (b) to the entrant's message strategy, we find that $\hat{\sigma}_m^*$ specified in $(\hat{\sigma}^*)$ is an optimal strategy. If $\pi^1(\mathbf{q}) + B$ is greater than $\underline{\delta}(q_1) + w_0$, $P^*(\mathbf{q}) = 1$ in the original equilibrium by Lemma 1 (b); thus $\hat{\sigma}_m^*(q_2|\mathbf{q}) = 1$ in the new strategy. If smaller, $P^*(\mathbf{q}) = 0$ thus the entrant sends only messages in $M_0(q_1)$ in the original equilibrium; so does he in the new strategy $\hat{\sigma}_m$. If equal, the new strategy gets him send $\tilde{q}_2 = q_2$ with probability $P^*(\mathbf{q}) \in [0, 1]$ and messages in $M_0(q_1)$ with the same probability as in the original strategy. Applying Lemma 1 (b) to $\hat{\sigma}_m$, we find that $\hat{\sigma}_m^*$ specified in $(\hat{\sigma}^*)$ is an optimal strategy.

Pooling-stay case Consider $q_1 \in Q_1$ such that $M_0(q_1) = \emptyset$ and $M_1(q_1) \neq \emptyset$. In the original equilibrium, the Bayesian belief and the strategy profile in the perturbation satisfy for any $q_2 \in Q_2, m \in M_1(q_1)$

$$\sigma_2^k(q_2)\sigma_m^k(m|q_1, q_2) = \mu^k(q_2|q_1, m) \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_1, q_2').$$

Since $\sum_{m \in M_1(q_1)} \sigma_m^k(m|q_1, q_2) = 1$, we have

$$\sigma_2^k(q_2) = \sum_{m \in M_1(q_1)} \left\{ \mu^k(q_2|q_1, m) \sum_{q_2' \in Q_2} \sigma_2^k(q_2')\sigma_m^k(m|q_1, q_2') \right\}.$$

Fix $\tilde{q}_2 \in Q_2$ arbitrarily. Because $\hat{\mu}^*(q_2|q_1, \tilde{q}_2) = \hat{\sigma}_2^*(q_2)$, we have at the limit

$$\hat{\mu}^*(q_2|q_1, \tilde{q}_2) = \hat{\sigma}_2^*(q_2) = \sum_{m \in M_1(q_1)} \left\{ \mu^*(q_2|q_1, m) \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_1, q_2') \right\}.$$

If $\hat{\mu}^*(q_2|q_1,\tilde{q}_2) > 0$, then there exists some $m \in M_1(q_1)$ such that

$$\mu^*(q_2|q_1,m) \sum_{q_2' \in Q_2} \sigma_2^*(q_2')\sigma_m^*(m|q_1,q_2') > 0, \quad \therefore \mu^*(q_2|q_1,m) > 0.$$

Applying Lemma 1 (a) to $\sigma_a^*(\cdot|q_1,m)$, we find

$$\pi^1(q_1,q_2) + B \geq 0,$$

since $\sigma_a^*(1|q_1,m) = 1$ for $m \in M_1(q_1)$. This holds for all q_2 with $\hat{\mu}^*(q_2|q_1,\tilde{q}_2) = \sigma_2^*(q_2) > 0$. Now applying it to $\hat{\sigma}_a^*(\cdot|q_1,m)$, we have $\hat{\sigma}_a^*(1|q_1,\tilde{q}_2) = 1$. So $\hat{\sigma}_a^*$ specified in $(\hat{\sigma}^*)$ is actually an optimal strategy.

Because any message $\tilde{q}_2 \in Q_2$ induces the same additional lender's strategy $\sigma_a^*(1|q_1,\tilde{q}_2) = 1$ and the same total payment $\underline{\delta}(q_1)$, all messages in $\hat{M}(q_1) = Q_2$ are indifferent for the entrant. So $\hat{\sigma}_m^*$ specified in $(\hat{\sigma}^*)$ is an optimal strategy.

Pooling-exit case In this case the strategy and belief profile is the same as the original. So the optimality in the original equilibrium retains in the new equilibrium.

Therefore we establish the consistency of belief $\hat{\mu}^*$ with $\hat{\sigma}^*$ and the sequential rationality of the strategy profile $\hat{\sigma}^*$, and thus the profile $(\hat{\mu}^*, \hat{\sigma}^*)$ is a sequential equilibrium under the quasi-direct mechanism $\hat{\mathcal{C}}$ with the message space $\hat{M}$. □

B Proof of Theorem 2

Proof. Let the additional lending strategy σ_a^* be

$$\sigma_a^*(1|q_1,\tilde{q}_2) = 1 \text{ for any } q_1 \in Q_1^S, \tilde{q}_2 \in Q_2^S(q_1),$$
$$\sigma_a^*(1|q_1,m_0) = 0 \text{ for any } q_1 \in Q_1,$$

and the entrant's messaging strategy σ_m^* be

$$\sigma_m^*(q_2|q_1,q_2) = 1 \text{ if } q_1 \in Q_1^S, q_2 \in Q_2^S(q_1),$$
$$\sigma_m^*(m_0|q_1,q_2) = 1 \text{ otherwise},$$
$$\sigma_m^*(\tilde{q}_2|q_1,q_2) = 0 \text{ for any } \tilde{q}_2 \neq q_2.$$

The (pure) capacity strategies $\mathbf{q}^*$ as (16) and these additional lender's strategy σ_a^* and messaging strategy σ_m^* constitute a (post-entry) sequential equilibrium with belief μ^*. If $q_1 \in Q_1^S$, for each $q_2 \in Q_2$

$$\mu^*(q_2|\tilde{q}_2, q_1) = I(q_2, \tilde{q}_2) \quad \text{for each } \tilde{q}_2 \in Q_2^S(q_1),$$

$$\mu^*(q_2|m_0,q_1) = \begin{cases} (1 - I^S(q_2|q_1))/(\#Q_2 - \#Q_2^S(q_1)) & \text{if } q_2^* \in Q_2^S(q_1), \\ I(q_2, q_2^*) & \text{otherwise.} \end{cases}$$

Here $I(q_2, q_2')$ is the indicator function for $q_2 = q_2'$ and $I^S(q_2|q_1)$ is the one for $q_2 \in Q_2^S(q_1)$. If $q_1 \notin Q_1^S$, for each $q_2 \in Q_2$

$$\mu^*(q_2|m_0,q_1) = I(q_2, q_2^*).$$

The belief μ^* is consistent with the sequence of perturbed strategy profiles σ^k such as

$$\sigma_i^k(q_i) := \frac{1}{\sqrt{k}\#Q_i} + \left(1 - \frac{1}{\sqrt{k}}\right) I(q_i, q_i^*) \quad \text{for each } q_i \in Q_i,\ i = 1, 2, ;$$

$$\sigma_m^k(\tilde{q}_2|q_1,q_2) := \frac{1}{2k\#Q_2^S(q_1)} + \left(1 - \frac{1}{k}\right) I(q_2, \tilde{q}_2) \quad \text{for each } q_1 \in Q_1^S, q_2 \in Q_2, \tilde{q}_2 \in Q_2^S(q_1);$$

$$\sigma_m^k(m_0|q_1,q_2) := \frac{1}{2k} I^S(q_2|q_1) + \left(1 - \frac{1}{2k}\right)(1 - I^S(q_2|q_1)) \quad \text{for each } q_1 \in Q_1^S, q_2 \in Q_2;$$

$$\sigma_a^k(1|q_1,\tilde{q}_2) := 1 - 1/k, \quad \sigma_a^k(1|q_1,m_0) := 1/k \quad \text{for each } q_1 \in Q_1^S, \tilde{q}_2 \in Q_2^S(q_1);$$

$$\sigma_m^k(m_0|q_1,q_2) := 1 \quad \text{for each } q_1 \notin Q_1^S, q_2 \in Q_2;$$

$$\sigma_a^k(1|q_1,m_0) := 1/k \quad \text{for each } q_1 \notin Q_1^S.$$

The strategy profile σ^k induces the Bayesian belief μ^k as follows. If $q_1 \in Q_1^S$ and

$\tilde{q}_2 \in Q_2^S(q_1) \setminus \{q_2^*\}$, the belief $\mu^k(\cdot|\tilde{q}_2, q_1)$ is

$$
\begin{aligned}
&\mu^k(q_2|\tilde{q}_2, q_1) \\
&:= \frac{\sigma_2^k(q_2)\sigma_m^k(\tilde{q}_2|q_1, q_2)}{\sum_{q_2' \neq \tilde{q}_2, q_2^*} \sigma_2^k(q_2')\sigma_m^k(\tilde{q}_2|q_1, q_2') + \sigma_2^k(\tilde{q}_2)\sigma_m^k(\tilde{q}_2|q_1, \tilde{q}_2) + \sigma_2^k(q_2^*)\sigma_m^k(q_2^*|q_1, \tilde{q}_2)} \\
&= \frac{(\sqrt{k}\#Q_2)^{-1}(2k\#Q_2^S(q_1))^{-1}}{\frac{\#Q_2 - 2}{\sqrt{k}\#Q_2 \cdot 2k\#Q_2^S(q_1)} + \frac{1}{\sqrt{k}\#Q_2}\left(\frac{1}{2k\#Q_2^S(q_1)} + 1 - \frac{1}{k}\right) + \left(\frac{1}{\sqrt{k}\#Q_2} + 1 - \frac{1}{\sqrt{k}}\right)\frac{1}{2k\#Q_2^S(q_1)}} \\
&= \left[\#Q_2 - 2 + \left\{1 + 2(k-1)\#Q_2^S(q_1)\right\} + \left\{1 + (\sqrt{k} - 1)\#Q_2\right\}\right]^{-1} \\
&= \left[\sqrt{k}\#Q_2\#Q_2 + 2(k-1)\#Q_2^S(q_1)\right]^{-1}, \\
&\mu^k(q_2^*|\tilde{q}_2, q_1) = \frac{\sigma_2^k(q_2^*)}{\sigma_2^k(q_2)}\mu^k(q_2|\tilde{q}_2, q_1) \\
&= \left\{1 + (\sqrt{k} - 1)\#Q_2\right\}\left[\sqrt{k}\#Q_2\#Q_2 + 2(k-1)\#Q_2^S(q_1)\right]^{-1}
\end{aligned}
$$

for each $q_2 \neq \tilde{q}_2, q_2^*$, and

$$
\begin{aligned}
\mu^k(\tilde{q}_2|\tilde{q}_2, q_1) &= 1 - \sum_{q_2 \neq \tilde{q}_2} \mu^k(q_2|\tilde{q}_2, q_1) \\
&= 1 - (\sqrt{k}\#Q_2 - 1)\left[\sqrt{k}\#Q_2\#Q_2 + 2(k-1)\#Q_2^S(q_1)\right]^{-1}.
\end{aligned}
$$

If $q_1 \in Q_1^S$ and $q_2^* \in Q_2^S(q_1)$, $\mu^k(\cdot|q_2^*, q_1)$ is given by

$$
\begin{aligned}
&\mu^k(q_2|q_2^*, q_1) \\
&:= \frac{\sigma_2^k(q_2)\sigma_m^k(\tilde{q}_2|q_1, q_2)}{\sum_{q_2' \neq q_2^*} \sigma_2^k(q_2')\sigma_m^k(\tilde{q}_2|q_1, q_2') + \sigma_2^k(\tilde{q}_2)\sigma_m^k(\tilde{q}_2|q_1, \tilde{q}_2)} \\
&= \frac{(\sqrt{k}\#Q_2)^{-1}(2k\#Q_2^S(q_1))^{-1}}{\frac{\#Q_2 - 1}{\sqrt{k}\#Q_2 \cdot 2k\#Q_2^S(q_1)} + \left(\frac{1}{\sqrt{k}\#Q_2} + 1 - \frac{1}{\sqrt{k}}\right)\left(\frac{1}{2k\#Q_2^S(q_1)} + 1 - \frac{1}{k}\right)} \\
&= \left[\#Q_2 - 1 + \left\{1 + (\sqrt{k} - 1)\#Q_2\right\}\left\{1 + 2(k-1)\#Q_2^S(q_1)\right\}\right]^{-1} \\
&= \left[\sqrt{k}\#Q_2\#Q_2 + 2(k-1)\#Q_2^S(q_1)\left\{1 + (\sqrt{k} - 1)\#Q_2\right\}\right]^{-1}
\end{aligned}
$$

for each $q_2 \neq q_2^*$, and

$$
\begin{aligned}
&\mu^k(q_2^*|q_2^*, q_1) = 1 - \sum_{q_2 \neq q_2^*} \mu^k(q_2|q_2^*, q_1) \\
&= 1 - (\#Q_2 - 1)\left[\sqrt{k}\#Q_2\#Q_2 + 2(k-1)\#Q_2^S(q_1)\left\{1 + (\sqrt{k} - 1)\#Q_2\right\}\right]^{-1}.
\end{aligned}
$$

If $q_1 \in Q_1^S$ and $q_2^* \in Q_2^S(q_1)$, $\mu^k(\cdot|m_0, q_1)$ is given by

$$
\begin{aligned}
&\mu^k(q_2|m_0, q_1) \\
=&\sigma_2^k(q_2)\sigma_m^k(m_0|q_1, q_2) \\
&\times \left[\sigma_2^k(q_2^*)\sigma_m^k(m_0|q_1, q_2^*) + \sum_{q_2' \in Q_2^S(q_1)\setminus\{q_2^*\}} \sigma_2^k(q_2')\sigma_m^k(m_0|q_1, q_2') + \sum_{q_2' \notin Q_2^S(q_1)} \sigma_2^k(q_2')\sigma_m^k(m_0|q_1, q_2')\right]^{-1} \\
=&\frac{(\sqrt{k}\#Q_2)^{-1}\left(1 - \frac{1}{2k}\right)}{\left(\frac{1}{\sqrt{k}\#Q_2} + 1 - \frac{1}{\sqrt{k}}\right)\frac{1}{2k} + \frac{\#Q_2^S(q_1)-1}{\sqrt{k}\#Q_2 \cdot 2k} + \frac{\#Q_2 - \#Q_2^S(q_1)}{\sqrt{k}\#Q_2}\left(1 - \frac{1}{2k}\right)} \\
=&\left[\frac{\left\{1 + (\sqrt{k}-1)\#Q_2\right\} + (\#Q_2^S(q_1) - 1)}{2k-1} + \#Q_2 - \#Q_2^S(q_1)\right]^{-1} \\
=&\left[\frac{2k + \sqrt{k} - 2}{2k-1}\#Q_2 - \frac{2k-2}{2k-1}\#Q_2^S(q_1)\right]^{-1}
\end{aligned}
$$

for each $q_2 \notin Q_2^S(q_1)$,

$$
\begin{aligned}
\mu^k(q_2|m_0, q_1) &= \frac{1/2k}{(1-1/2k)}\left[\frac{2k + \sqrt{k} - 2}{2k-1}\#Q_2 - \frac{2k-2}{2k-1}\#Q_2^S(q_1)\right]^{-1} \\
&= \left[(2k + \sqrt{k} - 2)\#Q_2 - (2k-2)\#Q_2^S(q_1)\right]^{-1}
\end{aligned}
$$

for each $q_2 \in Q_2^S(q_1) \setminus \{q_2^*\}$ and

$$
\begin{aligned}
\mu^k(q_2^*|m_0, q_1) &= \frac{(\sqrt{k}\#Q_2)^{-1} + 1 - 2k^{-1}}{(\sqrt{k}\#Q_2)^{-1}}\left[(2k + \sqrt{k} - 2)\#Q_2 - (2k-2)\#Q_2^S(q_1)\right]^{-1} \\
&= \{1 + (\sqrt{k}-1)\#Q_2\}\left[(2k + \sqrt{k} - 2)\#Q_2 - (2k-2)\#Q_2^S(q_1)\right]^{-1}.
\end{aligned}
$$

If $q_1 \in Q_1^S$ and $q_2^* \notin Q_2^S(q_1)$, $\mu^k(\cdot|m_0, q_1)$ is given by

$$
\begin{aligned}
&\mu^k(q_2|m_0, q_1) \\
=&\sigma_2^k(q_2)\sigma_m^k(m_0|q_1, q_2) \\
&\times \left[\sum_{q_2' \in Q_2^S(q_1)} \sigma_2^k(q_2')\sigma_m^k(m_0|q_1, q_2') + \sum_{q_2' \notin Q_2^S(q_1)\cup\{q_2^*\}} \sigma_2^k(q_2')\sigma_m^k(m_0|q_1, q_2') + \sigma_2^k(q_2^*)\sigma_m^k(m_0|q_1, q_2^*)\right]
\end{aligned}
$$

for each $q_2 \in Q_2$. If $q_1 \notin Q_1^S$, $\mu^k(q_2|m_0, q_1) = \sigma_2^k(q_2)$ for each q_2.

Take the limits as $k \to \infty$. If $q_1 \in Q_1^S$ and $\tilde{q}_2 \in Q_2^S(q_1) \setminus \{q_2^*\}$, $\mu^k(\tilde{q}_2|\tilde{q}_2, q_1) \to 1 - (\sqrt{k}\#Q_2 - 1) \cdot 0 = 1$. If $q_1 \in Q_1^S$ and $q_2^* \in Q_2^S(q_1)$, $\mu^k(q_2^*|q_2^*, q_1) = 1 - (\#Q_2 - 1) \cdot 0 = 1$.

If $q_1 \in Q_1^S$ and $q_2^* \in Q_2^S(q_1)$, $\mu^k(q_2|m_0, q_1) \to (\#Q_2 - \#Q_2^S(q_1))^{-1}$ for each $q_2 \notin Q_2^S(q_1)$. If $q_1 \in Q_1^S$ and $q_2^* \notin Q_2^S(q_1)$, $\mu^k(q_2|m_0, q_1) \to I(q_2, q_2^*)(1 - I^S(q_2|q_1))/(0+0+1) = I(q_2, q_2^*)$ for each $q_2 \in Q_2$. If $q_1 \notin Q_1^S$, $\mu^k(q_2|m_0, q_1) = \sigma_2^k(q_2) \to \sigma_2^*(q_2)$ for each q_2. Therefore, $\mu^k \to \mu^*$.

First of all, $\sigma_a^*(1|m, q_1) = 0$ is always at least one of the best response regardless of the belief $\mu^*(\cdot|m, q_1)$, according to Lemma 1 (a). So $\sigma_a^*(1|m_0, q_1) = 0$ is optimal.

If $q_1 \in Q_1^S$, any message $\tilde{q}_2 \in Q_2^S(q_1)$ implies $q_2 = \tilde{q}_2$ and $\pi^1(q_1, q_2) + B = \pi^1(q_1, q_2) + (\bar{V} + w_0) \geq 0$. So $\sigma_a^*(1|\tilde{q}_2, q_1) = 1$ is the best response. As the total repayment followed by any of such messages $\tilde{q}_2 \in Q_2^S(q_1)$ is the same as $\delta(\tilde{q}_2|q_1) = B - w_0$, $\sigma_m^*(q_2|q_2, q_1) = 1$ is the entrant's optimal messaging strategy, as long as $\pi^1(q_1, q_2) + \bar{V} + w_0 \geq 0$, i.e., $q_2 \in Q_2^S(q_1)$. Otherwise, the entrant chooses m_0 to exit.

In period 1, the entrant optimizes q_1 given $q_2 = q_2^*$. The capacity $q_1 \in Q_1^S$ eventually yields $\pi^1(q_1, q_2^*) + B + \bar{V} - \delta(q_2^*|q_1) = \pi^1(q_1, q_2^*) + \bar{V} + w_0$; $q_1^* \in Q_1^S$ is the best among Q_1^S by its definition in (16a). Any capacity $q_1 \notin Q_1^S$ eventually leads the entrant to exit, since $M(q_1) = \{m_0\}$; it gives him payoff of $B + \bar{V} - (B + w_0) = \bar{V} + w_0$; this is worse than q_1^* and thus q_1^* is the optimal among all $q_1 \in Q_1$, as long as $\pi^1(\mathbf{q}^*) \geq 0$.

The incumbent could get the entrant to exit by setting q_2 such that $\pi^1(q_1, q_2^*) + \bar{V} + w_0 < 0$, i.e., $q_2 \notin Q_2^S(q_1^*)$. Since $q_1^* \in Q_1^S$ and thus $Q_2^P(q_1^*) \subset Q_2^S(q_1^*)$, such q_2 yields the predatory profit $\pi^2(0, q_2)$ smaller than $\pi^2(\mathbf{q}^*)$. Any capacity size $q_2 \in Q_2^S(q_1^*)$ yields the duopoly profit $\pi^2(q_1^*, q_2)$, which is maximized at $q_2 = q_2^*$. So q_2^* is the optimal. □

C Proof of Corollary 1

Before the proof, let us rephrase Corollary 1 a bit more formally:

Corollary 1. *Suppose Assumptions 1,2 and 3. Let the entrant's start-up liquidity w_0 plus private value $\bar{V}$ of the asset be smaller than the maximal plausible predatory loss in the benchmark equilibrium $\bar{L}^P(q_1^\dagger)$:*

$$\bar{V} + w_0 < \bar{L}^P(q_1^\dagger) = -\pi^1(q_1^\dagger, \bar{q}_2^P(q_1^\dagger)). \tag{20}$$

Then, compared with the benchmark $\mathbf{q}^\dagger$, the entrant's capacity q_1^* shrinks while the incumbent's q_2^* expands:

$$q_1^* < q_1^\dagger, \quad q_2^* > q_2^\dagger.$$

Proof. If q_i can take any real number, the incumbent's maximal plausible predatory capacity

$\hat{q}_2^P(q_1)$ is characterized by

$$\pi^2(0, \hat{q}_2^P(q_1)) = \pi^2(q_1, \hat{q}_2^{BR}(q_1)) \text{ and } \pi_2^2(0, \hat{q}_2^P(q_1)) < 0,$$

where $\hat{q}_2^{BR}(q_1)$ is the incumbent's optimal capacity without predation:

$$\hat{q}_2^{BR}(q_1) = \arg\max_{q_2 \in \mathbb{R}} \pi^2(q_1, q_2).$$

The maximal plausible predatory loss $\hat{L}^P(q_1)$ is then

$$\hat{L}^P(q_1) := -\pi^1(q_1, \hat{q}_2^P(q_1)).$$

Differentiating the above equations with regard to q_1, we have

$$\frac{d\hat{L}^P}{dq_1}(q_1) = -\pi_1^1(q_1, \hat{q}_2^P(q_1)) - \pi_2^1(q_1, \hat{q}_2^P(q_1))\frac{d\hat{q}_2^P}{dq_1}(q_1), \tag{21}$$

and

$$\pi_2^2(0, \hat{q}_2^P(q_1))\frac{d\hat{q}_2^P}{dq_1}(q_1) = \pi_1^2(q_1, \hat{q}_2^{BR}(q_1)).$$

The latter implies

$$\frac{d\hat{q}_2^P}{dq_1}(q_1) = \frac{\pi_1^2(q_1, \hat{q}_2^{BR}(q_1))}{\pi_2^2(0, \hat{q}_2^P(q_1))} > 0$$

by $\pi_1^2(\cdot) < 0$ and $\pi_2^2(0, \hat{q}_2^P(q_1)) < 0$.

By Assumption 3, the equilibrium capacity $\mathbf{q}^*$ is the solution $\hat{\mathbf{q}}^*$ of (16) when q_i can take any real number. With the Lagrange multiplier λ of the non-predation condition (13), $\mathbf{q}^*$ should satisfy the first order conditions:[24]

$$\pi_1^1(\mathbf{q}^*) = \lambda\frac{d\hat{L}^P}{dq_1}(q_1^*), \quad \pi_2^2(\mathbf{q}^*) = 0.$$

If $\lambda = 0$ then the equilibrium capacity $\mathbf{q}^*$ was exactly the same as the benchmark $\mathbf{q}^\dagger$. Because

[24] To determine the entrant's best response uniquely, it is sufficient that the marginal revenue minus the implicit marginal "cost of financing" decreases to the entrant's capacity size q_1, given the incumbent's q_2:

$$\frac{\partial}{\partial q_1}\left[\pi_1^1(\mathbf{q}) - \lambda\left\{-\pi_1^1(q_1, \hat{q}_2^P) - \pi_2^1(q_1, \hat{q}_2^P)\frac{d\hat{q}_2^P}{dq_1}(q_1)\right\}\right]$$
$$= \pi_{11}^1(\mathbf{q}) + \lambda\left\{\pi_{11}^1(q_1, \hat{q}_2^P) + 2\pi_{12}^1(q_1, \hat{q}_2^P)\frac{d\hat{q}_2^P}{dq_1}(q_1) + \pi_{22}^1(q_1, \hat{q}_2^P)\frac{d^2\hat{q}_2^P}{dq_1^2}(q_1)\right\} < 0$$

This is guaranteed if π^1 is linear in q_2 ($\pi_{22}^1(\cdot) \equiv 0$), for example $\pi^1(\mathbf{q}) = (a - q_1 - q_2)q_1 - c_1q_1$.

the non-predation condition (13) is assumed to be violated at $\mathbf{q}^\dagger$ and thus $\mathbf{q}^\dagger$ cannot be a solution of (16), the Lagrange multiplier λ must be positive.

Substituting (21) into the above FOC yields

$$\pi_1^1(\mathbf{q}^*) - c_1 = \lambda \left[-\pi_1^1(q_1^*, \hat{q}_2^P(q_1^*)) - \pi_2^1(q_1^*, \hat{q}_2^P(q_1^*)) \frac{d\hat{q}_2^P}{dq_1}(q_1^*) \right].$$

Since $\pi_{12}^1(\cdot) \leq 0$ and $q_2^* = \hat{q}_2^{BR}(q_1^*) < \hat{q}_2^P(q_1^*)$, we have

$$\pi_1^1(\mathbf{q}^*) \geq \pi_1^1(q_1^*, \hat{q}_2^P(q_1^*)).$$

Combining these two expressions, we obtain

$$(1+\lambda)\pi_1^1(\mathbf{q}^*) \geq -\lambda \pi_2^1(q_1^*, \hat{q}_2^P(q_1^*)) \frac{d\hat{q}_2^P}{dq_1}(q_1^*).$$

As $\lambda > 0$, $\pi_2^1(\cdot) < 0$ and $d\hat{q}_2^P/dq_1 > 0$, this implies

$$\pi_1^1(\mathbf{q}^*) > 0.$$

That is, the entrant's response curve shifts downward (at least near the equilibrium $\mathbf{q}^*$). In contrast, that of q_2 obviously remains the same. We therefore conclude that the entrant's equilibrium capacity shrinks, while the incumbent's expands, compared with the benchmark.

□

D Commitment to stochastic additional lending policy

Here we consider the case where the initial lender can commit himself to a stochastic policy on additional lending $\sigma_a : M \otimes Q_1 \to [0,1]$ and the incumbent knows this stochastic policy precisely. We focus on the truth-telling quasi-direct mechanism $M(q_1) = Q_2^S(q_1) \cup M_0$ and induce the non-predation condition that is quite less restrictive than that (13) under deterministic commitment or no commitment. Our goal is to prove the following theorem.

Theorem 5. *Suppose a quasi-direct mechanism with truth telling. Further, assume that the initial lender can commit himself to a stochastic additional lending policy* $\sigma_a : Q_2^S \times Q_1 \to [0,1]$ *and the incumbent knows this policy precisely. Then the entrant can prevent the* *predation while setting the benchmark capacity $q_2^\dagger$ if and only if for all $q_2 \in (q_2^\dagger, \bar{q}_2^P(q_1^\dagger))$*

$$\bar{V} + w_0 \geq -\pi^1(q_1^\dagger, q_2) - \frac{\pi^2(\mathbf{q}^\dagger) - \pi^2(q_1^\dagger, q_2)}{\pi^2(0, q_2) - \pi^2(\mathbf{q}^\dagger)} \pi^1(\mathbf{q}^\dagger) \tag{22}$$

Proof. First, as we want to see the non-predation equilibrium with the benchmark capacity $\mathbf{q}^\dagger$, the lender should commit to keep the entrant stay with probability one if he sends the message of the benchmark capacity $q_2^\dagger$ of the incumbent given $q_1^\dagger$: $\sigma_a(1|q_2^\dagger, q_1^\dagger)$. So the incentive compatibility for the truth telling at $q_2 = q_2^\dagger$ is now

$$\pi^1(\mathbf{q}^\dagger) - \delta(q_2^\dagger) + B - w_0 \geq \sigma_a(1|q_2) \left(\pi^1(\mathbf{q}^\dagger) - \delta(q_2) + B - w_0\right)$$

for all $q_2 \in Q_2^S$. (Here and henceforth we omit $q_1^\dagger$ from the arguments in δ, σ, Q_2^S.) Like the proof in the deterministic/uncommitted additional lending case, the lender's participation condition implies $\delta(q_2^\dagger) \geq B - w_0$, and the entrant's limited liability constraint implies $\delta(q_2) \leq B + \pi^1(q_1^\dagger, q_2) + \bar{V}$. With them, the incentive compatibility above reduces to

$$\pi^1(\mathbf{q}^\dagger) \geq \sigma_a(1|q_2) \left(\pi^1(\mathbf{q}^\dagger) - \pi^1(q_1^\dagger, q_2) - \bar{V} - w_0\right),$$

or equivalently

$$\bar{V} + w_0 \geq -\pi^1(q_1^\dagger, q_2) + \left(1 - \frac{1}{\sigma_a(1|q_2)}\right) \pi^1(\mathbf{q}^\dagger).$$

The incumbent will not try predation if the equilibrium profit with $\mathbf{q}^\dagger$ is higher than the expected profit given the survival probability $\sigma_a(1|q_2)$ at any plausible predatory capacity $q_2 \in (q_2^\dagger, \bar{q}_2^P(q_1^\dagger)]$:

$$\pi^2(\mathbf{q}^\dagger) \geq \sigma_a(1|q_2)\pi^2(q_1^\dagger, q_2) + (1 - \sigma_a(1|q_2))\pi^2(0, q_2),$$

or equivalently

$$\sigma_a(1|q_2) \geq \frac{\pi^2(0, q_2) - \pi^2(\mathbf{q}^\dagger)}{\pi^2(0, q_2) - \pi^2(q_1^\dagger, q_2)}.$$

Combining the last equations in the last two paragraphs, we obtain the equation (22). □

Comparing the condition (22) with the non-predation condition (13), it is obvious that the requirement on the start-up capital plus the mortgaged asset $\bar{V} + w_0$ is reduced by $\pi^1(\mathbf{q}^\dagger)\{\pi^2(\mathbf{q}^\dagger) - \pi^2(q_1^\dagger, q_2)\}/\{\pi^2(0, q_2) - \pi^2(\mathbf{q}^\dagger)\}$. Actually we have non-trivial restriction (positive lower bound) on $\bar{V} + w_0$ only if there exists $q_2 \in (q_2^\dagger, \bar{q}_2^P(q_1^\dagger))$ s.t.

$$-\pi^1(q_1^\dagger, q_2) > \frac{\pi^2(\mathbf{q}^\dagger) - \pi^2(q_1^\dagger, q_2)}{\pi^2(0, q_2) - \pi^2(\mathbf{q}^\dagger)} \pi^1(\mathbf{q}^\dagger) > 0;$$

otherwise, the condition is satisfied with any non-negative $\bar{V} + w_0$.