Trade and Inequality in a Directed Search Model with Firm and Worker Heterogeneity

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Abstract

This paper integrates the insight that exporting firms are typically more productive and employ higher skilled workers into a directed search model of the labor market. The model generates a skill premium as well as residual wage inequality among identical workers. A trade liberalization will cause a reallocation of workers both within and across industries. The within industry reallocation increases the skill premium, increases residual inequality for low-skilled workers, and decreases residual inequality for high-skilled workers. The across industry reallocation induces the well-known Stolper-Samuleson effect. The calibrated model generates results consistent with the prior literature examining the effect of the Canada-U.S. Free Trade Agreement on the Canadian labor market: a significant decrease in employment in manufacturing, but only a small change in wages.

Keywords: Directed Search, Inequality, International Trade

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1 Introduction

Over the past three decades, both international trade and income inequality have increased, in developed and developing economies alike. While it is challenging to establish a causal link between trade and inequality, most empirical evidence is suggestive of a (weak) positive relationship.¹ However, the predictions of the standard work-horse model of international trade, the Heckscher-Ohlin (HO) model, are inconsistent with these findings. The HO model predicts an increase in the skill premium in the high-skill abundant country and a decrease in the skill premium in the low-skill abundant country (Stolper-Samuelson effect). One of the explanations for the absence of a strong Stolper-Samuelson effect in the data is the relative lack of worker reallocation between industries, the channel stressed in the HO model.²

An alternative mechanism for international trade to affect the skill premium is through the interplay between skill-biased technology and firm level adjustments: a trade liberalization induces an expansion of the largest and most productive firms (exporters), which also use more advanced technologies. This within industry reallocation of workers towards firms with a higher skill intensity increases the relative demand for high-skilled workers and, in turn, increases the skill premium, see e.g. Yeaple (2005).³

In this paper, I develop a directed search model of the labor market which features both within and across industry reallocation of workers to explore the relationship between international trade and income inequality. In addition to inequality among different workers (skill premium), the directed search framework also parsimoniously generates inequality among identical workers (residual inequality). In this way, it differs from much of the literature on trade and inequality, which typically focuses on either the skill premium or residual inequality.⁴

The directed search model follows Shi (2002, 2005) and Shimer (2005). Firms post wages to attract workers, workers observe all posted wages and then select a firm to which to apply.

¹See Feenstra and Hanson (2003) for a U.S.-focused study and Goldberg and Pavcnik (2007) for as survey of developing countries.
²See references in Goldberg and Pavcnik (2007).
³This mechanism is also present in Davidson et al. (2008a, 2008b) who study the effect of offshoring in a random search model with skill-biased technology. Recently, Costinot and Vogel (2010) and Burstein and Vogel (2012) have stressed this channel outside a search model of the labor market.
⁴Noticeable exceptions are Davidson et al. (2008a, 2008b).
Workers can only apply to one vacancy at a time and cannot coordinate amongst each other. The competitive nature of this mechanism requires that all vacancies offer identical workers the same expected utility. When workers apply, they trade off the probability of being chosen for a position and the wage they would receive should they be hired. Firms take this trade-off as given when they post their wage offers. Firms differ in their productivity and more productive firms have a stronger incentive to fill their vacancies and hence try to attract more applicants by posting higher wages for all types of workers. Differences in firm productivity thus translate into residual inequality in a directed search framework.

The production technology is skill biased – worker skill and firm productivity are complementary, so skilled workers have a comparative advantage at high productivity firms. In order to export, firms have to incur a fixed cost. As a result, only the most productive firms engage in exporting and expand their labor force (as in Melitz, 2003). Because of the complementarity between firm productivity and worker skill, this increases the relative demand for high-skilled workers. Firms in the middle of the productivity distribution now have a lower probability of attracting high-skilled workers and attract more low-skilled workers instead, while firms in the bottom of the productivity distribution cut employment. This within industry reallocation increases the skill premium and the wage dispersion among low-skilled workers, but has an ambiguous effect on inequality among high-skilled workers.

As in the standard HO framework, the model also allows for an across industry reallocation, which may increase or decrease both types of inequality, depending on the relative skill intensities of the comparative advantage industry (the Stolper-Samuelson effect). Consequently, the ambiguous effects of trade on income inequality found in the empirical literature are consistent with the model. The model also highlights an important general equilibrium effect that is often ignored in empirical assessments of trade reforms. Exploiting the country’s comparative advantage increases aggregate output and hence demand for the non-traded goods and services. This will cause a reallocation of resources out of traded industries and into non-traded industries and, as a consequence, overall employment in traded industries (importing and exporting combined) falls.

To further investigate these empirical implications of the model and compare the model’s pre-
dictions to a well-studied event, I calibrate the model to the Canadian labor market and study the impact of the Canada-U.S. Free Trade Agreement (FTA). The model’s predictions of the impact of the FTA on the Canadian labor market are in line with prior empirical work (e.g. Beaulieu, 2000, Trefler, 2004, Townsend, 2007): a noticeable decrease in overall manufacturing employment and a neglectable increase in inequality. The within and across industry effects have counteracting effects on inequality and are of similar strength in the case of the FTA.

This paper is related to the recent literature that combines search models of the labor market with trade models to study the link between inequality and trade. The work closest to the one in this paper is by Davidson et al. (2008a), who develop a model in which heterogeneous firms and workers match in a frictional labor-market. In that model, just as in the one industry variant of the model in this paper, opening the economy to international trade causes an expansion of high-tech firms, which in turn increases the skill premium and may lower residual inequality among high-skilled workers. However, in their model, workers and firms are randomly matched which potentially gives rise to multiple equilibria; in only one of these equilibria do high-skilled workers match with low technology firms, giving rise to residual inequality for that group. A different approach is taken by Helpman et al. (2010), who develop a model in which ex-ante identical workers are randomly matched with heterogenous firms. Workers receive unobserved productivity shocks and more productive firms screen workers more, giving rise to within-group inequality. In that setting opening the economy to trade increases inequality as it induces firm to screen more. In the symmetric mixed-strategy equilibrium of the directed search model used in this paper, both types of workers always match with a range of different firms, so there is always residual inequality among both groups.

The paper consists of three main sections. Section 2 lays out the static version of the model and section 3 extends the static model to a dynamic environment. Section 4 provides an illustrative

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5See Zhu and Trefler (2005), Davis and Harrigan (2007), Epifani and Gancia (2008), Egger and Kreickemeier (2009), Artuc et al. (2010), and Amiti and Davis (2012) for other approaches to studying the distributional consequences of international trade.

6Using a similar model, Davidson et al. (2008b) find that high-skill outsourcing has the opposite effect.

7The only other application of directed search in a trade setting I am aware of is King and Stähler (2011), who embed a directed search model into a standard general equilibrium trade model to study the effect of trade on unemployment, but do not address the question of inequality.
example of how the model can be used to study trade liberalization in the form of the Canada-U.S. Free Trade Agreement and section 5 concludes.

2 One Sector Static Model

2.1 Environment

In this section, I present a simple one-sector, partial-equilibrium model to illustrate the within sector skill-upgrading effect that will be at play in the full model in section 3. The model is a small open economy version of the directed search framework in Shi (2002) and Shimer (2005).

The economy consists of a large number of risk-neutral workers, with the total size of the labor force normalized to 1. A fraction \( \bar{h} \) of these workers is high-skilled (denoted \( h \)) and a fraction \( (1-\bar{h}) \) low-skilled (denoted \( u \)). There is a large number of firms, \( N \), that differ in their productivity, denoted \( s \geq 1 \). In order to produce, a firm must hire a worker. The technology is skill-biased: if a firm hires a low-skilled worker, its output is \( z \), independent of the firm’s productivity. However, if a firm hires a high-skilled worker, it produces \( sz \):

\[
y(s, i) = \begin{cases} 
  sz & \text{if } i = h \\
  z & \text{if } i = u
\end{cases}
\]

The labor market is frictional in the sense that firms and workers cannot coordinate their actions. Instead, firms and workers play the following three-stage game: after entering and observing their productivity, all firms post their offered wage for each skill level and their selection rule in case multiple workers apply to the vacancy. Individual workers observe all posted wages and make their application decisions. Lastly, firms select a worker out of the pool of applicants in accordance with the announced selection rule and begin producing output. Unmatched firms and workers produce nothing.

2.2 The Worker’s Problem

Workers cannot coordinate among each other as to which worker applies to which vacancy. Following the literature, I impose that identical workers use identical application strategies and
focus on symmetric, mixed strategy equilibria. Workers can condition their application probabilities on the type of firm (or vacancy), but not on the identity of the firm. So, all workers with the same skill level will apply to identical vacancies with the same probability. Consequently, it is possible that 2 workers apply to the same vacancy, while another vacancy receives no applications.

Workers choose their application strategy to maximize their expected wage taking the firms' offers and other workers' strategies as given. Let \( q(s, i) \) denote the expected number of type \( i \in \{h, u\} \) workers at a firm with productivity \( s \), the queue length. Assume that any firm that receives an application from both high and low-skilled workers prefers the high-skilled workers (I will verify below that this is the firm's optimal strategy). If the firm receives two or more applications of the same, preferred type of worker, the firm randomly chooses one of them. Accordingly, the hiring probabilities are given by

\[
g(s, h) = \frac{1 - e^{-q(s, h)}}{q(s, h)} \quad (1)
\]

\[
g(s, u) = e^{-q(s, h)} \frac{1 - e^{-q(s, u)}}{q(s, u)} \quad (2)
\]

The expected wage at one vacancy is denoted \( U(s, i) = g(s, i)w(s, i) \). Let \( \overline{U}^i \) denote the prevailing market utility (expected wage) for a worker of type \( i \). Now consider the case where one type of vacancy offers a higher expected wage than all other vacancies. All workers will apply to this vacancy with probability 1, and the resulting hiring probability for each worker is zero, which is a contradiction. Similarly, any vacancy offering a lower expected wage than others will not receive any applicants. Hence, in equilibrium

\[
\overline{U}^i \geq g(s, i)w(s, i), \quad q(s, i) \geq 0; \quad i \in \{h, u\} \quad (3)
\]

with complementary slackness. The workers' optimal application strategy is thus

\[
q(s, i) \begin{cases} 
0 & \text{if } U(s, i) < \overline{U}^i \\
(0, \infty) & \text{if } U(s, i) = \overline{U}^i 
\end{cases} \quad (4)
\]
From (3), the worker’s trade-off becomes apparent: in order to obtain a high wage, the worker must accept a lower hiring probability. This trade-off gives rise to inequality between identical workers (residual inequality). In other words, the underlying firm heterogeneity translates into income inequality: more productive firms are more eager to hire and hence attract a larger expected number of applicants. This lowers the hiring probability for each applicant, forcing more productive firms to post higher wages.

### 2.3 The Firm’s Problem without Exporting

A firm with productivity $s$ chooses wages and queue length to maximize expected profits

$$
\pi(s) = (1 - e^{-q(s,h)}) (psz - w(s,h)) + e^{-q(s,h)} (1 - e^{-q(s,h)})(pz - w(s,u)),
$$

where $p$ denotes the price of one unit of output in terms of some numeraire good and $w(s,i)$ denotes the wage paid to a worker of type $i$ at firm type $s$. Because of anonymity, firms can condition their wage on the skill level of the worker, but not on the worker’s identity. Thus, wages depend only on worker type, firm productivity and firm industry.

Using the expected wage condition, (3), wages can be eliminated and the expected profits can be written in terms of queue lengths only

$$
\pi(s) = (1 - e^{-q(s,h)}) psz - q(s,h)U^h + e^{-q(s,h)} (1 - e^{-q(s,u)})pz - q(s,u)U^u. \tag{5}
$$

Taking first order conditions with respect to the queue length yields

$$
U^h \geq e^{-q(s,h)}psz - e^{-q(s,h)}(1 - e^{-q(s,u)})pz, \quad q(s,h) \geq 0 \tag{6}
$$

$$
U^u \geq e^{-q(s,h)}e^{-q(s,u)}pz, \quad q(s,u) \geq 0 \tag{7}
$$

with complementary slackness.

From (6) and (7), one can recognize three different types of firms: firms that only attract high-
skilled workers, firms that only attract low-skilled workers, and firms that attract both types of workers. Substituting (7) as equality into (6) gives

\[ U^h \geq U^u + e^{-q(s,h)pz(s - 1)}, \quad q(s, h) \geq 0. \]  

(8)

Using (8) and (7), I can solve for queue lengths and cutoff productivity levels:

\[
q(s, h) = \begin{cases} 
0 & \text{if } s < \bar{s}^a \\
\log(pz) + \log(s - 1) - \log(U^h - U^u) & \text{if } s \in [\bar{s}^a, \bar{s}^b], \\
\log(pz) + \log(s - 1) - \log(U^h) & \text{if } s > \bar{s}^b
\end{cases}
\]  

(9)

\[
q(s, u) = \begin{cases} 
\log(pz) - \log(U^u) & \text{if } s < \bar{s}^a \\
\log(U^h - U^u) - \log(s - 1) & \text{if } s \in [\bar{s}^a, \bar{s}^b], \\
0 & \text{if } s > \bar{s}^b
\end{cases}
\]  

(10)

where

\[
\bar{s}^a = \frac{U^h - U^u}{pz} + 1, \]  

(11)

\[
\bar{s}^b = \frac{U^h}{U^u}.
\]  

(12)

The expected queue length of low-skilled workers is constant for low productivity firms and then decreasing in firm productivity as the expected queue of high-skilled applicants becomes longer. Because low-skilled workers produce the same amount of output independent of the firm’s productivity, all firms that attract no high-skilled workers are effectively identical and hence attract the same number of low-skilled workers. Note that this implies that workers receive the same wage at all of these firms. For firms that also attract high-skilled workers, low-skilled workers serve as an “insurance” in case no high-skilled worker applies. However, as the expected number of high-skilled
applicants increases, it becomes increasingly less likely that a low-skilled worker is selected. From
the worker’s trade-off, (3), the lower hiring probability increases the wage the firm must promise;
once the queue of high-skilled workers becomes sufficiently long, the firm stops attracting low-skill
applicants.

For high-skilled workers, the expected queue length is increasing in firm productivity. This
is because more productive firms have a stronger incentive to fill a vacancy and produce output.
At the same time, in order to attract a long queue of workers, a firm must offer a high wage to
compensate for the resulting low hiring probability. Thus, only the most productive firms can afford
to attract a long queue of high-skilled applicants.

Now, I can verify that firms indeed prefer high-skilled workers. Solving (3) for wages and
substituting (6) and (7) for \( U^h \) and \( U^u \) gives

\[
w(s, h) = \frac{q(s, h)}{1 - e^{-q(s, h)}} e^{-q(s, h)} p z \left( (s - 1) + e^{-q(s, u)} \right) < p z \left( (s - 1) + e^{-q(s, u)} \right)
\]

\[
w(s, u) = \frac{q(s, u)}{1 - e^{-q(s, u)}} p z e^{-q(s, u)} > p z e^{-q(s, u)}
\]

From these two inequalities, it follows that

\[w(s, h) - p z > w(s, u) - p z.\]

The left-hand side of the inequality is the profit generated by hiring a high-skilled worker, while the
right-hand side is the profit generated by hiring a low-skilled worker. Since the former is strictly
larger than the latter, firms prefer to hire high-skilled workers.

2.4 The Firm’s Problem with Exporting

Now consider the problem of a firm with access to the world market at some fixed cost \( f^x \). If
the firm chooses to export, it will receive \( p^w > p \) per unit of output. The firm makes its export
decision after hiring a worker. A firm will export if the profits from exporting exceed the profits
from domestic sales, i.e. if \( p^w y(s, i) - f^x > p y(s, i) \). Consistent with the empirical evidence that the
majority of firms do not export, I focus on the case where $f^x$ is large relative to the difference in prices, so only firms in the top of the productivity distribution will find it profitable to export. This implies that firms that hire low-skilled workers will never export.\(^8\) Then the cut-off productivity level for exporting is given by

$$\bar{s}^c = \frac{f^x}{z(p^w - p)}. \quad (13)$$

Depending on the relative magnitude of $f^x$ and $(p^w - p)$, this productivity cut-off could be above or below the cut-off for firms trying to attract low-skill as well as high-skill workers, $\bar{s}^b$. However, for the remainder of the exposition in this section, I will focus on the case in which the exporting cut-off, $\bar{s}^c$, exceeds $\bar{s}^b$. The case where $\bar{s}^c < \bar{s}^b$ can be solved similarly.

The expected profits of a firm expecting to export if matched with a high-skill worker are given by

$$\pi(s) = (1 - e^{-q(s,h)}) (p^w s - f^x) - q(s,h) U^h + e^{-q(s,h)} (1 - e^{-q(s,u)}) p z - q(s,u) U^u. \quad (14)$$

Taking first order conditions and solving for the optimal queue lengths gives

$$q^x(s,h) = \begin{cases} 
0 & \text{if } s < \bar{s}^a \\
\log(pz) + \log(s - 1) - \log(U^h - U^u) & \text{if } s \in [\bar{s}^a, \bar{s}^b) \\
\log(pz) + \log(s) - \log(U^h) & \text{if } s \in [\bar{s}^b, \bar{s}^c) \\
\log(p^w zs - f^x) - \log(U^h) & \text{if } s \geq \bar{s}^c 
\end{cases} \quad (15)$$

$$q^x(s,u) = \begin{cases} 
\log(pz) - \log(U^u) & \text{if } s < \bar{s}^a \\
\log(U^h - U^x) - \log(s - 1) & \text{if } s \in [\bar{s}^a, \bar{s}^b] \\
0 & \text{if } s > \bar{s}^b 
\end{cases} \quad (16)$$

\(^8\)This is not only a natural assumption, but it is an outcome in the general equilibrium version of the model since some firms in the exporting industry must serve the domestic market.
2.5 Skill Upgrading

Comparing the autarky queue lengths for high-skilled workers (9) with those in the trade equilibrium (15), it is apparent that the demand for high-skilled workers is increased. Substituting the exporting condition (13) as equality into (15) shows that a firm that is indifferent between being an exporter and a non-exporter has the same number of expected high-skilled applicants under autarky and under trade. However, for firms with productivity above the cut-off level, the expected queue length of high-skilled workers exceeds that which the firm would attract in autarky.

The increase in demand for high-skilled workers at exporting firms causes their expected wage to increase in order to restore the equilibrium in the labor market, i.e. \( \bar{U}_h \) increases. Consequently, firms in the middle of the productivity distribution reduce their demand for high-skilled workers, and increase their demand for low-skilled workers. Firms at the bottom of the productivity distribution shorten their queue length of low-skilled applicant (see Figure 1. for a numerical example). The overall effect is an increase in the expected wage for both types of workers and an increase in the relative expected wage, \( \bar{U}_h / \bar{U}_l \).

This reallocation of workers towards more productive firms also implies that the fraction of output produced by low productivity firms falls. Firms that attract only low-skilled workers are less likely to see their vacancy filled, while firms in the middle of the productivity distribution are now more likely than before to hire a low-skilled worker, reducing their output.

While the effect of the reallocation of workers within an industry on the skill premium is relatively easy to see, the effect on within-group dispersion is less obvious. Within an industry, high-skilled workers are now more concentrated and low-skilled workers are more dispersed, measured by the range of firm productivities. For high-skilled workers, this implies that the range of wages decreases, but how this translates into changes in residual inequality depends on the exact distribution of firm productivities. For low-skilled workers, residual inequality increases as a larger number of low-skilled workers is now employed by firms also attracting high-skilled workers and paying low-skilled workers a higher wage. The overall change in within-group wage dispersion across all groups (as commonly measured in empirical studies) depends on the exact distribution
of workers across firms and will be explored in the quantitative section.\footnote{In a similar model, Shi (2005) shows that skill-biased technological progress leads to a decrease in inequality among workers who can use the new technology.}

While the within-industry reallocation produces clear results for the skill premium, opening up to trade will also cause a reallocation of workers across industries. Taken on its own, the across industries reallocation causes a Stolper-Samuelson effect; the worker type that is more intensively hired in the exporting industry benefits. Depending on the relative skill intensities of the industries and the magnitude of worker reallocation, this effect may intensify the within industry effect on the skill premium or work in the opposite direction. This ambiguity in the direction of the effect of trade on inequality suggests that the question of trade and inequality is ultimately an empirical one. To evaluate the model quantitatively, the next section embeds this static environment into a dynamic multi-industry small open economy model.

3 Dynamic Model

Time is discrete and lasts forever. Workers are risk neutral and discount the future at rate $\beta \in (0, 1)$. As before, workers can be of two types, high-skill ($h$) and low skill ($u$), and the fraction of high skill workers is denoted by $\bar{h}$. There is a large number of inactive firms that can become active by paying a fixed entry cost $f^e$. After entering, firms draw their productivity $s$ from a distribution $F(s)$ and then seek to recruit workers (described in detail below). As before, firms have access to a skill biased technology where output depends on the type of worker hired:

$$y(s, i) = \begin{cases} sz & \text{if } i = h \\ z & \text{if } i = u. \end{cases}$$

The economy consists of $J$ industries, each producing a distinct good $Q_j$. These goods are aggregated into a final consumption good $C$ using a Leontief technology:

$$C = \min \{ \sigma_1 Q_1, \ldots, \sigma_J Q_J \},$$

where $\sigma_j$ is the relative weight of industry $j$. All firms within an industry produce a homogenous
good, i.e. $Q_j$ is the sum of all output by firms in industry $j$. The first $J - 1$ industries produce tradable goods whereas industry $J$ produces a non-tradable (service) good. This service good also serves as the numeraire in the economy, i.e. $p_J = 1$. The economy is small relative to the rest of the world and takes the import/export price of tradeable goods, $p_j^w$, as given. Exporting is costly; in order to export, firms must pay a per-period fixed cost $f^x$ (in terms of the numeraire good).

3.1 The Labor Market

As in the static model, workers and firms interact in a frictional labor market. After entering, firms post vacancies; if a vacancy is filled, the firm and worker start producing immediately and stay together until either the worker terminates the match or the match gets separated exogenously at rate $(\delta_h, \delta_u)$. After separation, the firm exits and the worker enters the pool of unemployed and can start searching immediately. If the vacancy is not filled, the firm exits the market and returns to the pool of potential entrants. Neither the worker nor the firm can search while matched. In the presence of long-term matches between workers and firms, the distribution of these matches becomes a state variable of the economy; let $\mu$ denote this distribution of firm-worker matches.

Instead of posting a wage, the firm posts an output sharing rule, i.e. the firm promises to pay the worker a fraction $\alpha \in (0, 1)$ of the net output for the duration of the match. As with wages in the static model, the output share can be conditioned on the worker’s type but not the worker’s identity. The share remains constant throughout the match and does not depend on the state of the economy.\footnote{While this assumption increases the tractability of the problem substantially, in a stochastic environment it may give rise to an inefficiency: as a result of a shock, either the firm or the worker may like to unilaterally break the match although the joint surplus of the match exceeds the continuation value of separation. However, in this paper I focus exclusively on a deterministic steady state so this problem does not arise here.} However, the actual income of the worker may change as the value of the output changes; this is particularly the case when a firm switches its exporter status. The firm makes a forward-looking vacancy posting decision, i.e. if along the current path of the aggregate state the firm begins exporting in three periods, it would attract a longer queue of high-skilled workers than if the the current state were the steady state. All firms post in the same labor market; workers are not attached to any industry and are free to apply to any firm.

The Worker’s Problem. Workers observe all the posted vacancies and direct their search
to the vacancy with the highest expected payoff. I assume that firms that receive both types of
workers prefer to hire the high-skilled worker, which will be true in equilibrium.\footnote{As discussed in Shi (2005), such a lexicographic selection rule is problematic in a dynamic setting. The assumption of no on-the-job-search and disallowing separations by firms serves to keep this problem tractable.} The expected payoff for high- and low-skilled workers from applying to a vacancy with productivity $s$, expected queue lengths $q \equiv (q_h, q_u)$, and posted share $\alpha_i$ is given by

$$U_h(s, q_h, \alpha_h, \mu) = (1 - e^{-q_h}) J_h(s, \alpha_h, \mu) + \beta \left( 1 - \frac{(1 - e^{-q_h})}{q_h} \right) \bar{U}_h(\mu + 1)$$

and

$$U_u(s, q_u, \alpha_u, \mu) = e^{-q_u} \frac{(1 - e^{-q_u})}{q_u} J_u(s, \alpha_u, \mu) + \beta \left( 1 - e^{-q_u} \frac{(1 - e^{-q_u})}{q_u} \right) \bar{U}_u(\mu + 1),$$

respectively, where $\bar{U}_i(\mu)$ denotes the market utility of an unemployed worker of type $i$ and

$$J_h(s, \alpha_h, \mu) = \max \left\{ \alpha_h p z + \beta \left( (1 - \delta_h) J_{h+1}(s, \alpha_h, \mu + 1) + \delta_h \bar{U}_h(\mu + 1) \right) ; \bar{U}_h(\mu) \right\}, \quad \text{(19)}$$

$$J_u(s, \alpha_u, \mu) = \max \left\{ \alpha_u p z + \beta \left( (1 - \delta_u) J_{u+1}(s, \alpha_u, \mu + 1) + \delta_u \bar{U}_u(\mu + 1) \right) ; \bar{U}_u(\mu) \right\} \quad \text{(20)}$$

are the Bellman equations for a high/low-skilled worker employed by a firm with productivity $s$, receiving an output share of $\alpha$. Note that prices depend on the distribution of firm-worker matches, but for notational brevity this is suppressed in the above expressions.

Since workers cannot coordinate their application strategies, they play symmetric mixed strategies. As in the static version of the model, workers must be indifferent between all firms to which they apply. Thus, an individual worker of type $i$ has the following optimal application strategy:

$$q_i(s, \alpha_i, \mu) \begin{cases} 
= 0 & \text{if } U_i(s, q, \alpha_i, \mu) < \bar{U}_i(\mu) \\
\in (0, \infty) & \text{if } U_i(s, q, \alpha_i, \mu) = \bar{U}_i(\mu). 
\end{cases} \quad \text{(21)}$$

**The Firm’s Problem.** A firm with productivity $s$ chooses the queue lengths $q \equiv (q_h, q_u)$ and
output shares \( A \equiv (\alpha_h, \alpha_u) \) to maximize the value of an unfilled vacancy:\(^{12}\)

\[
V^*(s, \mu) = \max_{q,A} \left\{ (1 - e^{-q_h}) V^h(s, \alpha_h, \mu) + e^{-q_h} (1 - e^{-q_u}) V^u(s, \alpha_u, \mu) \right\}, \quad (22)
\]

subject to the satisfying the market utility constraint, (21), and with

\[
V^h(s, \alpha_h, \mu) = (1 - \alpha_h)psz + \beta (1 - \delta_h) V^h_{+1}(s, \alpha_h, \mu + 1) \quad (23)
\]

and

\[
V^u(s, \alpha_u, \mu) = (1 - \alpha_u)pz + \beta (1 - \delta_u) V^u_{+1}(s, \alpha_u, \mu + 1) \quad (24)
\]

denoting the value of a match with a high and low skilled worker, respectively.

This problem can be simplified by eliminating the worker’s share of output using the market utility condition (21). First, notice that (17) can be written as:\(^{13}\)

\[
U^h - \beta \bar{U}^h_{+1} = \frac{(1 - e^{-q_h})}{q_h} \alpha_h sz \left[ p + a_h p_{+1} + a_h^2 p_{+2} + \ldots \right]
\]

\[
+ \beta \delta_h \left( \bar{U}^h_{+1} + a_h \bar{U}^h_{+2} + a_h^2 \bar{U}^h_{+3} + \ldots \right) - \beta \bar{U}^h_{+1}, \quad (25)
\]

where \( a_h = \beta (1 - \delta_h) \); a corresponding expression can be derived for (18). Similarly, the value of a vacancy can be written as:

\[
V(s, q, A, \mu) = (1 - e^{-q_h}) (1 - \alpha_h)sz \left[ p + a_h p_{+1} + a_h^2 p_{+2} + \ldots \right]
\]

\[
+ e^{-q_h} (1 - e^{-q_u}) (1 - \alpha_u)z \left[ p + a_u p_{+1} + a_u^2 p_{+2} + \ldots \right]. \quad (26)
\]

After cross-multiplying (25) with \( q_h \) and rearranging, the resulting expression can be substituted

\(^{12}\)The industry index \( j \) is omitted if there is no risk of confusion.

\(^{13}\)To simplify notation, the state variable \( \mu \) is suppressed and subscripts \( +1, 2, \ldots \) are used to denote future periods.
into (26), which leads to

\[
V^*(s, \mu) = \max_q \left\{ (1 - e^{-q_h}) \left[ sz \left( p + a_h p_{+1} + a_h^2 p_{+2} + \ldots \right) \right. \right.
\]
\[
+ \beta \delta_h \left( U_h^{+1}_1 + a_h U_h^{+2}_2 + a_h^2 U_h^{+3}_3 + \ldots \right) - \beta U_h^{+1}_1 \right] \right.
\]
\[
+ e^{-q_h} (1 - e^{-q_u}) \left[ z \left( p + a_u p_{+1} + a_u^2 p_{+2} + \ldots \right) \right. \right.
\]
\[
+ \beta \delta_u \left( U_u^{+1}_1 + a_u U_u^{+2}_2 + a_u^2 U_u^{+3}_3 + \ldots \right) - \beta U_u^{+1}_1 \right] \right.
\]
\[
- \left. q_h \left( U_h^{+1}_1 - \beta U_h^{+1}_1 \right) - q_u \left( U_u^{+1}_1 - \beta U_u^{+1}_1 \right) \right\}. \tag{27}
\]

A similar expression can be derived for exporting firms by replacing \( p sz \) with \( p^w sz - f^e \). From the viewpoint of individual firms and workers, the sequence of future states of the economy, \{\mu_t\}_{t=1}^\infty\,\text{, and corresponding output prices and values of unemployment are given. Thus, the optimal queue lengths solve the first-order conditions associated with (27).}

From (27), an important property of the directed search equilibrium becomes apparent. The competitive nature of this mechanism leads to a maximization of the joint surplus of the firm and the applying workers: the terms in the first two lines of (27) are the expected match surplus created if the vacancy is filled with a high-skilled worker and similarly, the third and fourth lines are the expected match surplus if the vacancy is filled with a low-skilled worker. The last line is the cost of attracting workers, all but one of which will be turned down and remain unemployed for another period.

**Equilibrium in the Labor Market.** Free entry of firms implies that entering firms’ expected profits equal the cost of entering:

\[
\int_s V^*(s, \mu) dF(s) = f^e. \tag{28}
\]

Let \( n^e(\mu, j) \) denote the number of newly entering firms in industry \( j \) that satisfies (28) and let \( u_i \) denote the number of unemployed workers of type \( i \) at the beginning of the matching process. Then, the optimal queue lengths have to satisfy the following aggregate feasibility constraint:

\[
u_i = \sum_{j \in N} n^e(\mu, j) \int_s q_i(s, j) dF(s), \quad i \in (h, u). \tag{29}\]
The resulting number of new matches formed in the current period is given by:

\[
m_h(\mu) = \sum_{j \in N} n^e(\mu,j) \int_s (1 - e^{-q_h(s,j)}) dF(s) \quad (30)
\]

and

\[
m_u(\mu) = \sum_{j \in N} n^e(\mu,j) \int_s e^{-q_u(s,j)} (1 - e^{-q_h(s,j)}) dF(s). \quad (31)
\]

And next periods’ number of unemployed workers is given by:

\[
\begin{align*}
  u_{h,+1} &= u_h + \delta_h (\bar{\mu} - u_h) - (1 - \delta_h)m_h(\mu) \quad (32) \\
  u_{u,+1} &= u_u + \delta_u (1 - \bar{\mu} - u_u) - (1 - \delta_u)m_u(\mu) \quad (33)
\end{align*}
\]

### 3.2 The Goods Market

Given the Leontief technology for the production of the final consumption good, the demand for the first \(J - 1\) goods is given by

\[
Q^d_j = \frac{I/P}{\sigma_j}, \quad \forall j \in \{1, 2, ..., J - 1\},
\]

where \(I\) denotes total income and \(P = \sum_{j \in J} p_j/\sigma_j\) is the price of the final consumption good. The demand for the non-traded service good \(J\) consists of the demand derived from the final good, as well as the fixed cost of entry and exporting

\[
Q^d_J = \frac{I/P}{\sigma_J} + f e \sum_{j \in J} n^e(\mu,j) + f x \int_{s,j} \mathbb{I}_x(s,j)d\mu(h,j).
\]

\(\mathbb{I}_x(s,j)\) is an indicator variable equal to one if a firm with productivity \(s\) in industry \(j\) is an exporter:

\[
\mathbb{I}_x(s,j) = \begin{cases} 
1 & \text{if } p_j^u sz_j - f x > p_j sz_j \\
0 & \text{else.}
\end{cases} \quad (34)
\]

Since all firms in an industry produce a homogeneous product, the supply of good \(j\) by domestic
producers to the home market is the sum of all output produced by firms in industry $j$:

$$Q_{s,d}^j = \int_s s z_j (1 - \mathbb{I}_x(s,j)) d\mu(h,j) + \int_s z_j d\mu(u,j).$$

(35)

Note that (34) implies that the domestic price will be lower than the world price for exported goods, whereas the domestic price equals the world market price for imported goods. As a result, (35) is the total domestic supply for exported goods as there will be no imported goods in that industry. For importing industries, the total domestic supply is given by the sum of the domestic production and the imported quantity

$$Q_s^j = Q_{s,d}^j + Q_{s,m}^j,$$

where $Q_{s,m}^j$ is the imported quantity of good $j$.

4 Application: The Canada-U.S. Free Trade Agreement

In this section, I use the model to conduct an illustrative quantitative exercise studying the effects of the Canada-U.S. Free Trade Agreement (FTA), implemented on January 1, 1989. The agreement called for all tariffs between the two countries to be removed within 10 years. The FTA agreement lends itself as an illustration of the model because it was a clearly defined experiment without accompanying changes in labor market policies. In a developing country, trade reforms are often accompanied by other reforms, such those to promote FDI, which will lead to changes in the production technology – these reforms are themselves often skill-biased.

The labor market implications of the FTA have been subject to a series of empirical studies. Gaston and Trefler (1997) report that the FTA had only a small effect on employment and similarly Beaulieu (2000) finds that the FTA had a small negative effect on low-skill employment, but did not affect earnings. On the other hand, Trefler (2004) reports substantial employment losses – 12 percent for the industries that were previously most protected. He also finds a small increase in wages – about 3 percent – but no affect on income inequality. Lastly, Townsend (2007) reports a decline in the wages of those workers in sectors facing the largest tariff cuts: a 0.5% reduction in

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14 In 1988, the effective tariff on U.S. products was 16% (Trefler, 2004).
wages for each 1% reduction in tariffs. Focusing on plant productivity, Head and Ries (1999) find that the net-effect of the tariff cuts on output per plant is virtually zero, but that the FTA affected average plant-level output through industry composition. On the other hand, Trefler (2004) and Lileeva (2008) report that the FTA led to significant productivity gains in Canadian manufacturing, mostly as a result of the exit of low productivity plants.

4.1 Calibration

For the purpose of this exercise, I will treat the U.S. as the sole trading partner and Canada as a small open economy.\(^{15}\) First, the model is calibrated to match the Canadian labor market in the advent of the FTA. The FTA is introduced by exogenously lowering the price of import-competing sector products and increasing the price in the exporting sector by the change in the effective tariff rates. Then, the new steady state is computed.

To be consistent with the Leontief production structure that allows for no substitution between industries, I restrict the number of industries to three: export manufacturing, import-competing manufacturing and non-traded services, where import-competing industries are defined as the industries that saw a fall in net-exports between 1988 and 1996. The length of a period is one quarter and all empirical moments are for male workers. Overall, 12 parameters need to be calibrated: the fraction of skilled workers, \(\bar{h}\), the separation rates for both groups of workers, the shape of the productivity distribution distribution parameters, each industry’s productivity parameter, the fixed costs of entering and exporting, and the discount factor. Note that the productivity parameter \(z\) and expenditure parameter \(\sigma\) are not separately identified, so I set \(\sigma = 1\) for all industries.

Three parameters can be matched directly to their counterpart in the data: the fraction of high-skilled workers is set to the fraction of workers with a college or post-secondary certificate in 1988, \(\bar{h} = 0.304\) (Labor Force Historical Review 2000, Statistics Canada). The separation rates are set to match the average complete job length by educational attainment (1981-1989), \(\delta_h = 0.044\) and \(\delta_u = 0.086\) (Heisz, 1996). The firm productivity distribution is assumed to be exponential with parameter \(\lambda\). After normalizing the productivity of the non-traded sector to one, i.e. \(z_3 = 1\), and

\(^{15}\)71.6% of all Canadian trade in 1988 and 79.8% in 1998 was with the U.S. (Statistics Canada, CANSIM, Table 228-0003).
setting the discount factor to $\beta = 1.04^{-1/4}$, the remaining seven parameters are chosen to match the following moments:

- The college premium, defined as the difference in average log wages, adjusted for experience, between high-school and college graduates (Boudarbat et al., 2010).
- The skill distribution within industries (non-production workers are labelled as high-skilled and production workers as low-skilled; Trefler, 2004).
- The increase in the fraction of exporting firms (Baldwin and Gu, 2003).

Table 1 lists all targets and Table 2 the corresponding parameter values. The most striking result of the calibration is the low productivity dispersion.\textsuperscript{16} This is a result of the relatively low skill premium in Canada: a large productivity dispersion will cause a large skill premium because the most productive firms have the most incentive to fill their vacancy and will hence attract a relatively long queue and pay high wages. Furthermore, an increase in dispersion increases the mean of an exponential distribution, hence a more spread out distribution increases the average high-skill worker’s productivity.

The import-competiting sector is smaller (measured by employment) and more skill-intensive (measured by the ratio of non-production to production workers) than the exporting sector. The larger dispersion in the import-competeting sector is a consequence of the higher skill-intensity; only firms with a large enough productivity draw find it profitable to attract high-skill workers. Lastly, it is worth noting that overall employment in the export- and import-competeting industries is small: only a about 21% of the labor force is directly affected by the FTA.

4.2 Results

Using the calibrated parameters, I compute the pre-FTA steady state. The results are listed in the first column of Table 3. First, note that the employment rate for high-skill workers is close to 1.\textsuperscript{16} For example, Eaton et al. (2008) and Balistreri et al. (2011) estimated parameters for a Pareto distribution imply substantially more variation in firm productivity.
This is a result of the hiring rule that firms will always hire a high-skilled over a low-skilled worker. By matching the unemployment rate for low-skilled workers, the calibrated model overpredicts employment for high-skilled workers. Secondly, the model underpredicts within-group inequality. This is a consequence of low productivity dispersion. Inequality in the model arises from differences in job-finding probabilities. However, since the variation in the firm-/worker-level productivity is small, differences in queue lengths are small and the resulting wage dispersion is small. Also, the assumption that firm productivity has no impact on low-skilled worker output suppresses inequality among low-skilled workers.

The FTA is introduced by perturbing the pre-FTA prices by the amount of the effective tariff reductions (16%, Trefler, 2004). The FTA increases aggregate output and causes a sectorial relocation of workers, as shown in Table 4. The model predicts an increase in output of the final consumption good by a relatively modest 0.87%. That is, however, not too surprising since only a small part of the economy was directly affected by the FTA. Output and employment in import-competing industries shrink significantly, while the exporting industries expand. However, the net-effect on manufacturing employment is negative – employment in manufacturing is predicted to fall, both in absolute numbers as well as relative to non-traded services. Prior to the FTA, 21.2% of all employment was in manufacturing; after the FTA this fraction falls to 20.5%, which translates into a decrease in total manufacturing employment of 3.3%. This compares to a 5% drop in manufacturing employment that Trefler (2004) attributes to the FTA. However, it is important to stress that this loss in employment in manufacturing is not the consequence of import-competition “destroying jobs of hard-working Canadians,” as it might be termed in the popular press, but rather the result of an efficiency gain caused by increased trade. Due to the complementarity in the production of the final good, this efficiency gain in manufacturing causes an increase in demand for non-traded services, so employment in the non-traded service sector increases.

The import-competing industries are more high-skill intensive than the exporting industries, measured by the ratio of non-production to production workers, which causes the relative demand for high-skill workers to decrease. On the other hand, in order to export, a plant must be highly productive, which increases the relative demand for high-skilled workers. Since exporting is a rare
event, the skill-upgrading effect within the exporting sector is not very strong, so the sectorial composition effect dominates and the skill premium falls slightly. Similarly, the impact on within-group inequality is also small. Again, the import-competing sector is more dispersed than the exporting sector, so its shrinkage has a dampening effect on within-group inequality caused by the within industry reallocation. These predictions are in line with the empirical studies of the FTA cited above that mostly find a small effect on labor market outcomes. While trade volumes increased substantially, the fraction of the labor force affected by them was small and importing and exporting industries were not significantly different in terms of skill content.

The most substantial shortcoming of the model is the predicted productivity effect (as measured by the output per worker), shown in Table 4. The model predicts that productivity remains almost constant in all sectors, rather than increase as the empirical evidence suggests. This is a consequence of two modelling assumptions. First, the production technology assumes that the plant productivity has no impact on the output of the low-skilled worker. As Figure 1 shows, the probability of filling a vacancy is somewhat decreased for the plants with the lowest productivity draws and is increased for plants with high draws. However, since low-skilled workers have the same productivity everywhere, the fact that they are now matched with marginally better firms actually lowers the output for these firms (see Figure 2 for the probability that a firm that attracts both high and low-skilled applicants hires a low-skilled applicant). While the measured worker productivity does not increase, the average underlying plant productivity does. The second reason for this shortcoming is that labor is the only input in production. This prevents firms that export from increasing their capital stock and increasing output per worker, while technology upgrading might play a role in the data.\footnote{While there is no evidence of technology upgrading playing a role in the case of the FTA, technology upgrading and trade-induced skill-biased technological change have been mentioned as a source of increased inequality in developing countries (see for example Goldberg and Pavcnik, 2007).}

5 Conclusion

This paper presents a structural model of the labor market that generates equilibrium unemployment, income inequality between different skill groups, and income inequality between identical
workers. These features are generated by search frictions in the labor market combined with heterogeneity in firm productivity. The model is then used to study the impact of international trade on labor market outcomes such as unemployment and inequality.

The model highlights two avenues for trade to affect labor market outcomes. First, trade changes the distribution of workers within an industry: more productive firms increase their demand for high-skilled workers, forcing medium productivity firms to recruit more low-skilled workers instead of high-skilled ones and lowering the matching probability for low productivity firms. This increases the average firm productivity and the skill premium, while lowering the inequality among high-skilled workers and increasing it among low-skilled workers. Second, trade changes the distribution of workers across industries: not surprisingly, employment moves out of import-competing and into exporting industries. Depending on the relative skill intensities of the industries, the change in the industry composition may amplify or attenuate the effect of the within industry reallocation.

While it might be somewhat disappointing that the model does not predict a clear relationship between trade and inequality, it helps to explain the varied findings in the literature – ultimately, the characterization of this relationship is an empirical one. In the case of the Canada-U.S. FTA, the models predictions are in line with previous empirical work because, in the case of Canadian manufacturing, the within and across effects cancel each other out. In most developing countries, there is relatively little worker reallocation across industries due to the rigidity of their labor markets and the evidence tends to favor an increase in inequality, consistent with the within industry skill upgrading mechanism.

References


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### Table 1: Calibration Targets

<table>
<thead>
<tr>
<th>Data Moment</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Skill Premium</td>
<td>42.1%</td>
</tr>
<tr>
<td>Unemployment Rate Low Skill</td>
<td>8.7%</td>
</tr>
<tr>
<td>Fraction of Employment in Manufacturing</td>
<td>21.2%</td>
</tr>
<tr>
<td>thereof: Fraction in Exporting Industries</td>
<td>59.4%</td>
</tr>
<tr>
<td>Ratio Non-Production to Production</td>
<td>33.3%</td>
</tr>
<tr>
<td>Workers in Exporting Industries</td>
<td></td>
</tr>
<tr>
<td>Ratio Non-Production to Production</td>
<td>42.2%</td>
</tr>
<tr>
<td>Workers in Import-Competing Ind.</td>
<td></td>
</tr>
<tr>
<td>Increase in Fraction of Plants Exporting</td>
<td>9.6%</td>
</tr>
</tbody>
</table>

### Table 2: Calibration Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
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<td>$z_1$</td>
<td>6.60</td>
<td>$f^e$</td>
<td>2.49</td>
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<tr>
<td>$\lambda_2$</td>
<td>5.97</td>
<td>$z_2$</td>
<td>9.40</td>
<td>$f^x$</td>
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<tr>
<td>$\lambda_3$</td>
<td>5.29</td>
<td>$z_1$</td>
<td>1.00</td>
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</table>


### Table 3: Labor Market Outcomes

<table>
<thead>
<tr>
<th></th>
<th>Pre-FTA</th>
<th>Post-FTA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Avg. log-wage H-skill</td>
<td>0.1698</td>
<td>0.1693</td>
</tr>
<tr>
<td>Avg. log-wage L-skill</td>
<td>-0.2503</td>
<td>-0.2499</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>0.4201</td>
<td>0.4192</td>
</tr>
<tr>
<td>Std. Dev. H-skill Wages</td>
<td>0.0212</td>
<td>0.0213</td>
</tr>
<tr>
<td>Std. Dev. L-skill Wages</td>
<td>0.0173</td>
<td>0.0172</td>
</tr>
<tr>
<td>Employment Rate High-skill</td>
<td>99.5%</td>
<td>99.4%</td>
</tr>
<tr>
<td>Employment Rate Low-skill</td>
<td>91.3%</td>
<td>91.3%</td>
</tr>
</tbody>
</table>
### Table 4: Production Outcomes

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>Output (Final Consumption Good)</td>
<td>0.864</td>
<td>0.872</td>
</tr>
<tr>
<td>Productivity Exporting Industries</td>
<td>7.313</td>
<td>7.314</td>
</tr>
<tr>
<td>Fraction of Employment in Manufacturing</td>
<td>0.212</td>
<td>0.205</td>
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<tr>
<td>thereof: Fraction in Exporting Industries</td>
<td>0.594</td>
<td>0.760</td>
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<tr>
<td>Ratio Non-Production to Production Workers in Exporting Industries</td>
<td>33.3%</td>
<td>33.1%</td>
</tr>
<tr>
<td>Ratio Non-Production to Production Workers in Import-Competing Ind.</td>
<td>42.2%</td>
<td>42.4%</td>
</tr>
<tr>
<td>Fraction of Plants Exporting</td>
<td>–</td>
<td>9.6%</td>
</tr>
</tbody>
</table>
Figure 1: Vacancy Filling Rate
Figure 2: Vacancy Filling Rate, Low-Skill Workers