Risk and the Misallocation of Human Capital

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DETU Working Paper 21-03
August 2021

1301 Cecil B. Moore Avenue, Philadelphia, PA 19122

https://ideas.repec.org/s/tem/wpaper.html

Risk and the Misallocation of Human Capital

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June, 2021

Abstract

With risk-averse workers and uninsurable earnings shocks, competitive markets allocate too few workers to jobs with high earnings uncertainty. Using an equilibrium Roy model with incomplete markets we show that risky occupations are inefficiently small and hence talent is misallocated. We obtain analytical expressions for the compensation for risk in the labor market, and for the aggregate level of human capital and output. Misallocation is positively related to the correlation between a worker's abilities in different occupations. Quantitatively we find that market incompleteness can by itself generate permanent output and welfare losses in the order of one percent of GDP.

Key words: Misallocation, Human Capital, Occupations, Risk, Incomplete Markets, Frèchet, Roy Model.

JEL Classifications: E21 · D91 · J31.

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1 Introduction

Misallocation of human capital lowers productivity. Occupation or industry-specific human capital is an important feature of labor markets. For example, many technical, medical and legal occupations require knowledge in a narrowly defined field. It is rarely possible to work in such occupations without first obtaining occupation-specific skills and credentials through specialized training. At the same time, due to technological progress, international trade or urbanization, workers in certain occupations are subject to permanent earnings shocks that are hard to predict when making decisions about investing in skills training. The fear of high potential losses arises because there are no private insurance markets to hedge against these shocks. These shocks displace workers that are heavily invested in occupation- or industry-specific human capital.

In this paper we are the first to study how uninsurable permanent risk to a worker's human capital shapes the aggregate allocation of talent. Through the prism of a Roy model, we show that talent is misallocated in a *laissez faire* competitive equilibrium. Risk averse workers avoid risky occupations when insurance opportunities are absent, unless wages are sufficiently high. But at high wages the demand for workers is low and as a result risky occupations are inefficiently small. In our quantitative analysis we study cases in which shocks to workers' human capital are caused by policy (e.g. a trade reform) or by technological progress. We find that the misallocation caused *only* by market incompleteness produces permanent losses of around 0.6% of output. Our results shed new light on the cost of market incompleteness and they can inform policymakers when designing policies aimed at providing earnings or unemployment insurance for workers.

Our general equilibrium Roy model features a labor market where workers self-select into an occupation or industry based on their comparative and absolute advantages.[1] We assume that workers are risk-averse and human capital (for example acquired through specialized training) is specific to an occupation or industry. Workers' occupational choices determine both the level of output and the wage distribution in the economy. We compare the level of production efficiency in competitive equilibrium to an unconstrained planning problem which yields maximal output.

Our model features two occupations (without loss of generality) and the choice of a career is based on two factors: (i) a worker's inherent talents in each occupation, and (ii) each occupation's earnings uncertainty, measured by the variance of permanent shocks to earnings. The inherent talents of workers are modeled as draws from a Frèchet distribution. We allow for abilities to be correlated, which provides us with a tractable way of distinguishing between comparative and absolute advantages. One extreme case is that of perfectly correlated draws in which a worker's ability is the same across occupations (purely absolute advantage). The other extreme would be the case of independent draws (comparative advantage). The model's tractability allows us to obtain closed-form solutions for various outcomes of interest such as the allocation of workers, output, and the wage and earnings premia.[2] In addition, the tractability illustrates the mechanics of the interplay between abilities and risk in affecting allocations and output in a transparent way.

We measure misallocation by comparing output in a competitive equilibrium to output achieved by a social planner. The planner allocates workers across occupations based on their abilities in order to maximize output. Of course, the planner does not observe the shocks that workers receive once they have chosen an occupation. However, she can allocate consumption across workers after shocks are realized. The planner's allocation is identical to that obtained in a competitive equilibrium with risk-neutral workers. Although risk is compensated in the competitive equilibrium

[1]Except in the quantitative analysis; in the remainder of the paper we use the terms industry and occupation interchangeably.

[2]By a wage or an earnings premium we refer to the wage or earnings differential between the risky and the safe occupation.

— riskier occupations pay more — the planner allocates more workers to riskier occupations than the competitive equilibrium does, resulting in higher output. In a competitive equilibrium, the link between the marginal product of labor and the wage prevents the size of risky occupations from growing to the efficient level. At the efficient level, wages are too low to compensate for the extra risk borne by the individual.

As expected, misallocation is more severe the higher the workers' risk aversion. As risk aversion rises, entering the risky industry is less desirable and thus higher risk aversion exacerbates the costs of market incompleteness. We also find that the degree of misallocation is negatively related to the degree of comparative advantage. Independent draws (the extreme case of pure comparative advantage) imply a higher degree of selection because good abilities can only be used in one occupation. When the dependence is low for both abilities there is a higher likelihood that the worker has high ability in at least one occupation. Stronger selection – i.e. the sorting of workers into their higher ability by occupation – implies a better buffer against risk. Therefore, the absence of insurance markets matters less. As an additional result, we also provide a simple tax scheme that restores the planner's allocation.

Our quantitative analysis focuses on two questions that have received attention in the literature. We begin by calibrating the model to US data on earnings by industry. We use estimates of the variance of permanent shocks to earnings by industry and pick values for the rest of the parameters to match moments from the 2001 wave of the Survey of Income and Participation Program (SIPP). The earnings premium in the data is around 7% (after controlling for observables like education and age) which yields a risk aversion parameter of 2.9. We find that the maximum permanent output loss due exclusively to market incompleteness can be as high as 0.6%.

We also use our model to quantify the output losses associated with trade reforms. For this purpose, we make use of a number of studies that document a positive relationship between the degree of import penetration and the trade exposure of an industry with the volatility of worker's earnings. We take as given the increase in import penetration of the US manufacturing sector from 1991-2009. This rise in import penetration caused a reallocation of manufacturing workers. In light of our model, the increase in risk due to trade openness makes the tradable sector less attractive for future cohorts of workers. The increase in misallocation due only to the increase in risk in this period can plausibly be as large as 0.7 percentage points of total output. The corresponding decrease of manufacturing employment predicted by the model is as large as 4 percentage points (a third of that observed in US data).

1.1 Related Literature

Our paper connects several strands of literature in macroeconomics and labor economics. First, it relates to the macroeconomics literature on misallocation and development. As has been studied in many important papers (see e.g. Hsieh and Klenow (2009), Restuccia and Rogerson (2013), Lagakos and Waugh (2013), Lagakos, Mobarak, and Waugh (2018), Vollrath (2009), Midrigan and Xu (2014), Guner, Ventura, and Yi (2008)) the misallocation of factors of production across firms, sectors or regions within an economy is important to explain cross-country productivity differences. However, with some exceptions (see for example Vollrath (2014) and Hsieh, Hurst, Jones, and Klenow (2019), Buera, Kaboski, and Shin (2011), Bhattacharya, Guner, and Ventura (2013)) the misallocation of human capital has received much less attention. In our case we focus on one particular friction: the inefficiency of the competitive equilibrium allocation caused by incomplete markets. On one hand, this focus allows us to analyze the consequences of a specific friction whose existence is clear and present. On the other hand, we abstract from other important barriers to the allocation of workers to occupations and thus our results on misallocation may seem smaller than the ones reported for example in Hsieh, Hurst, Jones, and Klenow (2019). Furthermore, our analysis does not focus on some occupations or type of human capital since it includes all occupations and it's flexible enough to incorporate other aspects of worker occupational choice.

Our theoretical approach uses the insights of Roy (1951) and models workers' occupational choice under uncertainty. Thus, it connects to models of occupational choice used in macroeconomics and labor economics. Examples include Kambourov and Manovskii (2008, 2009), Jovanovic (1979), Miller (1984), Papageorgiou (2014), and Lopes de Melo and Papageorgiou (2016). We focus on the interplay between comparative advantages and risk in shaping worker' occupational choice and thus we complement their findings as well as the ones of Cubas and Silos (2017, 2020), Silos and Smith (2015), Hawkins and Mustre del Rio (2012), Dillon (2016), and Neumuller (2015). We differ from these papers by abstracting from career dynamics so we can obtain closed form solutions and a better characterization of the elements that affect the misallocation of human capital. One way to interpret our static model is to think of workers choosing a career (and the single period representing a worker's lifetime). Changes in risk due to, for example, technological progress affect different cohorts of workers at the time time they make their career choice (see for example Bart Hobijn and Vindas (2018)),

We think our application to trade reforms provides new insights to the literature trying to understand the effects of trade reforms on labor markets. Our framework does not incorporate international trade but it's flexible enough to measure the output losses associated with trade reforms when workers who are exposed to import competition are unable to insure against permanent shocks to their earnings. Thus, our work is also related to the work of Lyon and Waugh (2018), Lee (2020) and Traiberman (2019).

2 Model

The economy is populated by a continuum of workers of total mass equal to one who live for one period. They are endowed with a unit of time which they inelastically supply as labor. That unit of labor can be supplied in either of two occupations. One is risky (labeled as occupation R) and the other is safe (labeled as S).[3] Workers value the consumption of a final good produced according to the following CES technology.

$$Y = [\theta N_R^{\nu} + (1-\theta) N_S^{\nu}]^{1/\nu} \tag{1}$$

/noindent where N_R and N_S are the aggregate amount of efficiency units of labor in the risky and safe occupations, respectively, $0 < \theta < 1$ governs the share of each occupation in total output and ν is the elasticity of substitution between the two occupations.

Consumption of the final good is financed using labor earnings, as workers do not save and are born with zero wealth. Workers' preferences are described by a utility function of the constant relative risk aversion class. More specifically, given an amount of consumption c an individual ranks consumption levels c according to $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, with $\gamma > 1$.

Workers are endowed with a vector of occupation-specific abilities. These abilities can be thought as skills that are useful in a given occupation (for example, mathematical thinking for an engineer or physical strength for a construction worker). Some abilities may be innate but others can be the result of previously accumulated human capital. Nonetheless, we do not specify the origin of those abilities and we treat them as being predetermined at the time of the occupational choice. Abilities can be correlated across occupations and as a result some workers are likely to excel at several professions. In what follows, the vector of abilities is denoted by $\boldsymbol{X} = (X_R, X_S)$. We model the dependence between the two abilities through a Gumbel copula of two Fréchet random variables:

$$F(x_R, x_S) = Pr(X_R < x_R, X_S < x_S) = \exp\left\{ -\left[\sum_{i \in R,S} (T_i^{\alpha} x_i^{-\alpha})^{1/(1-\rho)} \right]^{(1-\rho)} \right\} \tag{2}$$

[3]Focusing on two occupations - one relatively risky and one relatively safe - is done only for simplicity. The framework can be easily generalized to an arbitrary number J of occupations.

The parameter T_i is the scale parameter. The parameter ρ controls the dependence across ability levels for a given worker and is bounded between 0 and 1. When ρ approaches 1 there is perfect dependence between the two ability draws. When it approaches zero, abilities are uncorrelated. The parameter α drives the dispersion and it is common to all abilities. We assume $\alpha > 2$ which ensures that the variance of the abilities distribution is finite. Given (2), the marginal distributions are standard univariate Fréchet with cdf

$$Pr(X_i < x_i) = \exp\left\{-\left(\frac{x_i}{T_i}\right)^{-\alpha}\right\} \tag{3}$$

We derive this resultn in section A of the Appendix. [4]

2.1 Occupational Choice and Sorting

Given a realization of $\boldsymbol{X} = (x_R, x_S)$, a worker opts for one of two alternative careers. In one of them, earnings are more uncertain and we assume that occupation R is the riskier one. The uncertainty is driven by shocks that alter a worker's ability to perform an occupation; shocks are distributed according to $F_i(y)$ for occupations $i = R, S$. We assume shocks are log-normal and have mean equal to one and $var(log(y_i)) = \sigma_i^2$. It is worth repeating—and this is what makes the problem interesting— that the occupational choice is conditional on the pre-determined abilities $\boldsymbol{X}$ but unconditional on the subsequent shock the worker experiences while on the job.

To formalize the occupational decision given $\boldsymbol{X}$ and the market prices for abilities in each occupation, w_R and w_S, the value of working in occupation i is denoted by $V_i(x_i, w_i)$ and it is equal to:

$$V_i(x_i, w_i) = \max_c \int_{y \in \mathbb{Y}} \frac{c^{1-\gamma}}{1-\gamma}\, dF_i(y) \tag{4}$$

[4] A similar approach is followed in Lind and Ramondo (2018). The authors augment a Ricardian trade model by using a multivariate max-stable Fréchet distributions to represent countries sectoral productivities.

$$\text{subject to } c \leq x_i\, e^y\, w_i$$

To determine the value of working in an occupation the worker needs to know the price of a unit of ability in that occupation, denoted by w_i and the worker's own pre-determined ability x_i. The prices of the skills, w_i, are determined in a competitive equilibrium but taken as given by the worker when choosing an occupation to enter. Once on the job, consumption is constrained by the total amount of ability $x_i\, e^y$ times its price w_i. As shocks y are stochastic with support $\mathbb{Y}$, the value of occupation i is given by the expected utility of consumption.

Among the two alternative careers, the worker picks the one with the highest value.

$$V(\boldsymbol{X}, w_R, w_S) = \max\{V_R(x_R, w_R), V_S(x_R, w_S)\} \tag{5}$$

Given that only two occupations are available, worker sorting in our environment is summarized by the share p_R of workers choosing the risky occupation.

Proposition 2.1 *The share of workers choosing occupation R,p_R, is given by*

$$p_R = \frac{T_R^{\frac{\alpha}{(1-\rho)}} |\Omega_R(w_R)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{\sum\limits_{i\in\{R,S\}} T_i^{\frac{\alpha}{(1-\rho)}} |\Omega_i(w_i)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \tag{6}$$

where $\Omega_i = \int_{y\in\mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma}\, dF_i(y)$.

To understand the result of proposition 2.1, note that a worker chooses the risky occupation when its value is larger than that of the safe occupation. To calculate the economy-wide fraction of workers that choose the risky occupation (p_R) we proceed as follows. Given market wages, for each value of the ability in the safe occupation we calculate the probability that the value of the risky occupation is larger. Averaging these probabilities using the distribution of abilities in the safe occupation yields the expression in the 2.1.

Note that the proportion of workers, everything else equal, increases with the wage rate. The proportion of workers also rises if T_R is higher (relative to T_S); a higher T_R raises the comparative advantage for occupation R raising the proportion of workers opting for that occupation.

Once we have found the probability that a worker chooses occupation R, and therefore the mass of workers performing occupation R, we need to characterize the abilities of the workers choosing this occupation to obtain total effective labor input.

Proposition 2.2 *The amount of efficiency units in occupation i is*

$$N_i = p_i \mathbb{E}(\tilde{x}_i) = p_i^{\frac{\alpha-(1-\rho)}{\alpha}} T_i \Gamma\left(1 - \frac{1}{\alpha}\right)$$

where $\mathbb{E}(\tilde{x}_i)$ *is the average ability of workers who choose occupation i (i.e. post-sorting).*

The result follows by first noting that $N_i = p_i \tilde{x}_i$, where $\tilde{x}_i$ is the average ability of a workers who choose occupation i.[5] In section C of Appendix we offer a proof of this proposition. In proving it we make use of a well-known result: if the marginal distributions of abilities pre-sorting is Fréchet, the post-sorting distribution of abilities is also Fréchet. More specifically, the post-sorting marginal distributions are Fréchet with shape parameter α and scale parameter $T_i p_i^{\frac{-(1-\rho)}{\alpha}}$. These parameters imply a mean ability for occupation i equal to $T_i p_i^{\frac{-(1-\rho)}{\alpha}} \Gamma(1 - \frac{1}{\alpha})$.

Note that $N_i = p_i^{\frac{-(1-\rho)}{\alpha}} p_i T_i \Gamma\left(1 - \frac{1}{\alpha}\right) = p_i^{\frac{-(1-\rho)}{\alpha}} \mathbb{E}(x_i)$ where $\mathbb{E}(x_i)$ is the average ex-ante ability (i.e. pre-sorting). Given that $\alpha > 2$ and $0 < \rho < 1$, it is easy to see that average skills of workers after sorting are higher than ex-ante average skills. This is the direct consequence of sorting given workers select based on their comparative advantage. When $\rho = 1$, i.e. when there is perfect dependence of abilities. As a result, there is no sorting on relative skills or comparative advantage. In this special case workers are equally skilled (or unskilled) in either occupation. Hence, the distributions of abilities pre- and post-sorting are identical.

[5]The shocks that workers experience after they have chosen an occupation are of mean equal to one so we can abstract from them when computing N_i.

2.2 The Competitive Equilibrium Allocation

A competitive equilibrium is a pair of employment levels (mass of efficiency units) N_R and N_S, and a pair of wages w_R and w_S, and an associated level of output Y_{CE}. The employment levels result from the solution to the workers' occupational choice problem, and wages are such that the labor market for each occupation clears. Since labor markets are perfectly competitive the wage rate in a given occupation equals the marginal product of employment of that occupation. Thus, using the expressions derived in 2.1 and 2.2, and the marginal products of each type of labor, we can derive closed form expressions for N_R and N_S. Substituting into the production function we obtain the following result.

Proposition 2.3 *The competitive equilibrium level of output Y_{CE} is given by*

$$Y_{CE} = \left\{ \theta T_R^{\nu} \left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha}((1-\rho)-\alpha)} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_S}{E_R}\right)^{\frac{\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\nu((1-\rho)-\alpha)}{\alpha}} + \right.$$

$$\left. (1-\theta) T_S^{\nu} \left[1 + \left(\frac{T_R}{T_S}\right)^{\frac{-\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha}((1-\rho)-\alpha)} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_R}{E_S}\right)^{\frac{\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{\nu((1-\rho)-\alpha)}{\alpha}} \right\}$$

$$\Gamma\left(1-\frac{1}{\alpha}\right)$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)}) = e^{(1-\gamma)(-\frac{\sigma_i^2\gamma}{2})}$

A detailed derivation of this result can be found in section D of Appendix. In a competitive equilibrium the level of output depends on two objects. First, on the shape of the production function, summarized by the share parameter θ and the elasticity of substitution across occupations $1/(1-\nu)$. Second, the level of efficiency units in each occupation. Efficiency units depend on the relative differences in the shape parameter T_i and on the proportion of workers that choose occupation i. This proportion is influenced by γ — the risk aversion coefficient — and its interaction with idiosyncratic risk. The ratio $(E_R/E_S)^{1/(1-\gamma)}$ rises as γ drops, making the riskier occupation relatively more attractive.

To sharpen the intuition we analyze the special case of a Cobb-Douglas technology.

$$Y_{CE} = T_R^{\theta}\left[\frac{\theta E_R^{\frac{1}{1-\gamma}}}{\theta E_R^{\frac{1}{1-\gamma}} + (1-\theta)E_S^{\frac{1}{1-\gamma}}}\right]^{\frac{\theta(\alpha-(1-\rho))}{\alpha}} T_S^{1-\theta}\left[\frac{(1-\theta)E_S^{\frac{1}{1-\gamma}}}{\theta E_R^{\frac{1}{1-\gamma}} + (1-\theta)E_S^{\frac{1}{1-\gamma}}}\right]^{\frac{(1-\theta)(\alpha-(1-\rho))}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right)$$

In the case of the Cobb-Douglas it is clear that the mass of workers in the risky occupation rises as risk aversion falls. The ratio $\frac{\theta E_R^{\frac{1}{1-\gamma}}}{\theta E_R^{\frac{1}{1-\gamma}}+(1-\theta)E_S^{\frac{1}{1-\gamma}}}$ rises as γ falls. Everything else constant, less risk aversion raises the fraction of workers in the risky occupation. Efficiency units in the R occupation also rise with the scale parameter T_R. The exponent $\theta(\alpha-(1-\rho))/\alpha$ increases with α for a given θ and ρ. A higher α fattens the upper tail of the abilities distribution, increasing average efficiency and raising output. The role of ρ is also clear from the expression. A higher value implies abilities for a given worker are more correlated, decreasing worker selection, lowering the amount of efficiency units, and therefore lowering output.

3 The Misallocation of Human Capital

We begin by solving for the efficient level of output. We then compare its value to the competitive equilibrium allocation. We finally relate misallocation — the difference in output between the two allocations — to the parameters of interest.

3.1 The Social Planner's Problem

In our framework, the efficient allocation is the one that maximizes output. We assume that a planner allocates workers across the two occupations after observing each worker's ability. Of course, the planner does not observe the shocks that workers receive once they begin work in an occupation. Therefore the planner makes the decision of where to allocate workers knowing only the ex-ante abilities (skills). Proposition 2.2 establishes the relationship between efficiency units in occupation i, N_i and its mass of workers, p_i.

Thus, we use it to solve the social planner's problem, which reduces to finding the masses of workers in occupations R and S, p_R^{SP} and p_S^{SP} that maximize output.

$$\max_{p_R^{SP}, p_S^{SP}} \left[\theta T_R^{\nu} \left(p_R^{SP}\right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} + (1-\theta) T_S^{\nu} \left(p_S^{SP}\right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} \right]^{1/\nu} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{7}$$

subject to,

$$p_R^{SP} + p_S^{SP} = 1 \tag{8}$$

By taking first order conditions we can solve for p_R^{SP} and p_S^{SP} in closed form. We then use 2.2 and the production function to obtain the efficient output, given by:

$$\begin{aligned} Y_{SP} = \Bigg[\theta T_R^{\nu} \left(\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} + \\ (1-\theta) T_R^{\nu} \left(\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}} \Bigg]^{1/\nu} \Gamma\left(1-\frac{1}{\alpha}\right) \end{aligned} \tag{9}$$

In Section F of the Appendix we provide more details about the derivation. It is again instructive to examine the much simpler Cobb-Douglas case to gain intuition about the role of the shape of the abilities distribution on the degree of misallocation. When the production function is Cobb-Douglas the efficient level of output is,

$$Y_{SP} = T_R^{\theta} \theta^{\frac{\theta(\alpha-(1-\rho))}{\alpha}} T_S^{(1-\theta)} (1-\theta)^{\frac{(1-\theta)(\alpha-(1-\rho))}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{10}$$

Note that when workers are risk neutral the level of output in competitive equilibrium is the same as that obtained by the social planner.[6] Risk does not matter for the allocation of resources, only the technology and the distributions of ex-ante abilities matter.

3.2 Discussion

Figure 1 helps to clarify the intuition for why market incompleteness misallocates workers across occupations resulting in Y_{SP} being larger than Y_{CE}. Suppose a simplified world with no ex-ante differences in abilities but with post-entry uninsurable risk. The figure shows aggregate output as a function of employment in occupation R, fixing the value of N_S for purposes of exposition. In a competitive equilibrium workers are indifferent between the two occupations. Occupation R is riskier than S and therefore its wage must compensate workers for bearing a higher risk. For the wage to be high enough the number of workers in the risky occupation has to be low since our technology exhibits diminishing marginal returns to either type of labor. This low level of employment corresponds to the value N_R^{CE} in the figure. The marginal product (the wage rate in equilibrium) is equal to the slope of the production function at that value. Thus, although risk is compensated in competitive equilibrium the resulting allocation does not maximize output. A social planner can increase output by reallocating workers across occupations resulting in an amount of employment in the risky occupation of N_R^{SP}. The corresponding marginal product is lower as shown by the flatter slope. Because employment in the planner's problem is set to maximize output, the competitive equilibrium leads to a risky occupation that is too small. With ex-ante abilities not all workers are indifferent between the two occupations in equilibrium, but the intuition for why talent is misallocated is the same.

How does misallocation change when preferences or abilities change?. In Figure

[6]To see this set γ equal to zero in the expression for Y_{CE}.

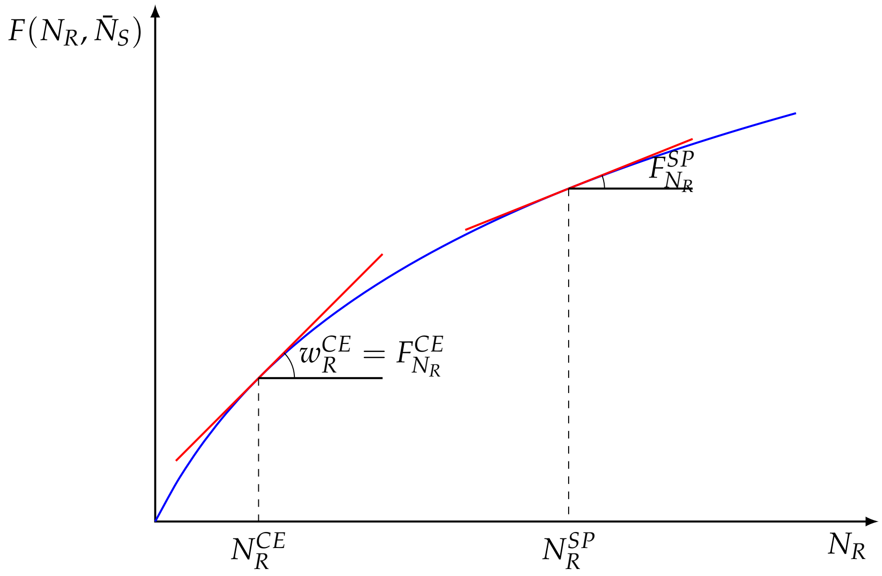


Figure 1: Risk, Compensating Differential and the Optimal Allocation

Notes: The figure shows the level of output and the allocation of workers in the risky industry (N_R) a simplified version of the model in both the laissez-faire competitive equilibrium (N_R^{CE}) and the social planner problem (N_R^{SP}).

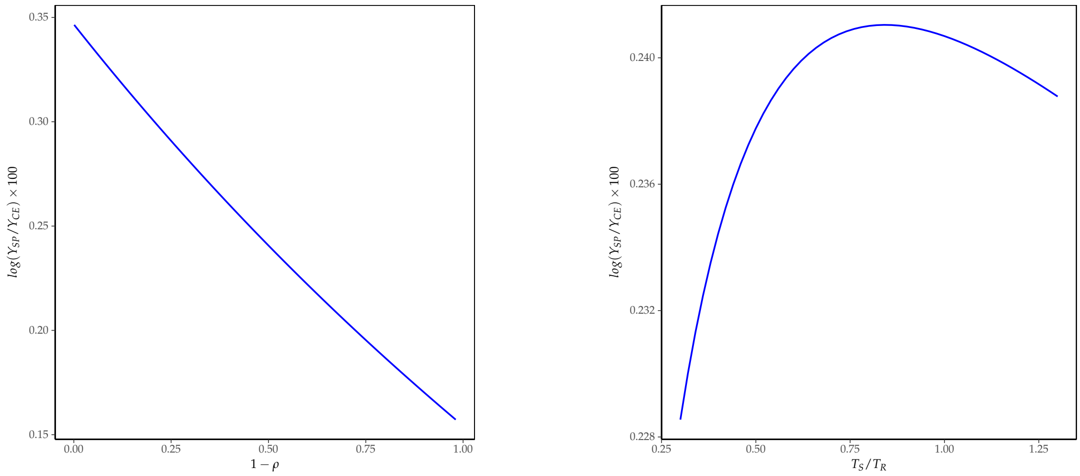
2 we plot the log of the ratio of Y_{SP}/Y_{CE} (in percentage terms) for different values of the parameters of interest.

We begin by analyzing misallocation for different values of ρ. Recall that ρ governs the dependence between the abilities of workers across occupations, also interpretable as the degree of comparative advantage. In other words, when ρ is close to one, if a worker is good at performing one occupation there is a high probability of being also good at the other occupation. We can think of ρ approaching one as the limiting case in which there is only one ability to perform both occupations or, just absolute advantage. The lower ρ is the lower the relative wage rate in the risky occupation. The reason is simple: when ρ is low there is more selection in equilibrium. It is always the case that fewer workers choose the risky occupation, but the lower the ρ the more selected they are and they have a higher mean ability conditional on choosing the risky occupation (and thus efficiency units). Due to the decreasing returns at the occupational level, the wage rate is lower. The higher mean ability provides insurance, which alleviates the negative effect of market incompleteness on output. Thus, as the figure shows, the lower the ρ the closer the competitive equilibrium allocation is to the optimal allocation.[7]

We also plot the degree of misallocation for different values of the ratio of the mean of ex-ante abilities T_S/T_R. As the inverted-U shape shows, for relatively low or high values of T_S/T_R the competitive equilibrium allocation is closer to the optimal allocation. When T_S/T_R is low, everything else equal, the abilities of occupation R workers are relatively high so even though fewer workers choose that occupation in equilibrium (compared to the social planner allocation) the mass of efficiency units is larger. Total output correspondingly gets closer to its optimal level. When T_S/T_R is high, everything else equal, the abilities of occupation R workers are relatively low. Therefore, occupation S is relatively more important for the planner to maximize output and so the optimal quantity of workers is relatively higher in that occupation. At the same time, occupation R is not that important and thus the gap between the number of workers in the competitive equilibrium allocation and the social planner allocation is not that consequential for the output gap.

Interestingly, in Section G of the Appendix we show that the efficient allocation can be recovered by a linear tax scheme that taxes relatively more the safe occupation.

[7] In Section E of the Appendix we illustrate this point in more detail as well as the compensation for risk premium in competitive equilibrium.



(a) Misallocation and $(1-\rho)$.

(b) Misallocation and T_S/T_R.

Figure 2: The two figures show how the degree of misallocation varies for different values of two parameters: (a) ρ and (b) T_S/T_R. Misallocation is measured by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}).

4 Quantitative Analysis

4.1 Labor Income Risk and the Misallocation of Workers Across US Industries

We use the results of Cubas and Silos (2017) and several other moments from US earnings data, to calibrate the model. Using the Survey of Income and Program Participation (SIPP) as the source of earnings data, Cubas and Silos (2017) decompose individual-level earnings in each US industry into a permanent and a transitory component. They estimate the variance of each component, reporting results for a total of 19 industries.

According to the estimates reported, industries vary greatly in their degree of permanent earnings volatility. We use their estimates and divide industries in two groups, the "risky" and the "safe" sector, according to the variance of the permanent component of earnings.[8]. The first group has a permanent variance of 0.00570

[8]The "risky" group includes Utilities, Finance, Nondurable Goods Manuf., Wholesale Trade, Com-

and the second a variance of 0.00399. Cubas and Silos (2017) also estimate a random walk process for the permanent component of earnings. Because our model is static, we assume a 40-year career for workers and thus multiply each variance by 40. This product represents the variance of the permanent component of earnings over a worker's life-cycle .

We need to calibrate the parameters of the copula, T_R, T_S, α and ρ, in addition to the aggregate technology parameters θ and ν, and the risk aversion parameter γ. Because in our general equilibrium framework mean earnings does not depend on the scale parameters of the Fréchet distribution (T_R and T_S) we fix them at a value of one. To calibrate α we employ the following procedure. Using the 2001 panel of the SIPP we estimate a fixed-effects regression for individual earnings controlling for age and time (the SIPP is a quarterly panel). We interpret the distribution of fixed effects as the distribution of worker productivities prior to experiencing shocks. Consistent with this interpretation we use the standard deviation of fixed effects across workers to calibrate α. Because α is the same for the two abilities distributions, we target the standard deviation of (log) abilities of the safe industry. The standard deviation of workers' fixed effects in the safe industry is 0.345 in the data. We estimate the share parameter θ in the aggregate technology by setting it so that the model delivers a share of workers in the risky industry of 75%, as observed in the data. Finally, to estimate the risk aversion coefficients we derive the expression for the compensation for risk in our environment. In our model, $EP = \left(\frac{E_R}{E_S}\right)^{\frac{1}{\gamma-1}}$. Section E of the Appendix contains the details on the derivation of this expression, but succinctly, it states that the ratio of average earnings across the two industries depends only on the risk aversion parameter γ and the two standard deviations of the earnings shocks. The earnings premium across the two industries is 6.75%, yielding a risk aversion coefficient of 2.92.

munication, Retail Trade, Medical Services, Transportation, Recreation and Entertainment, Construction, Durable Goods Manuf. and Other Services. The "safe" group includes Agriculture and Forestry, Social Services, Government, Hospitals, Business Services, and Personal Services.

Because we use the standard deviation of earnings to estimate α and the share of workers in the risky industry to estimate θ, we cannot separately estimate ρ. We opt to analyze the model by assuming a range of values for ρ (the minimum is 0.1 and the maximum is 1), recalibrating θ and α for each value of the dependency parameter.[9] Lastly, the parameter ν drives the elasticity of substitution across occupations. The literature lacks a clear reference for an estimate of this elasticity. We opt for a value of ν equal to 1/3 (an elasticity of 1.5). The implied elasticity of that value is halfway between the Cobb-Douglas case (ν equal to 0 or a unit elasticity of substitution) and an elasticity of substitution equal to 3 (or ν equal to 2/3) as used by Hsieh and Klenow (2009).

Figure 3 shows the difference between output in the competitive equilibrium and output in the social planner's problem for different values of $(1-\rho)$ and γ.

As in Figure 2, as ρ decreases the degree of misallocation decreases. The logic and intuition is the same: independent draws imply a higher degree of selection because high abilities can only be used in one occupation. When the dependence between abilities is low there is a higher likelihood that the worker has high ability in at least one occupation. The more selection – i.e. the higher average ability by occupation – implies a better buffer against risk and therefore the absence of insurance markets matters less. In addition, for a fixed ρ, the higher the value of the risk aversion parameter γ, the higher the degree of misallocation. As risk aversion rises, entering the risky industry is less desirable. Higher risk aversion exacerbates the costs of market incompleteness. These results provide a quantitatively plausible range of the level of misallocation. The minimum loss is 0.1% and the maximum loss is around 0.6% of output, permanently.

[9] This procedure delivers a range of values for θ between 0.698 and 0.716.

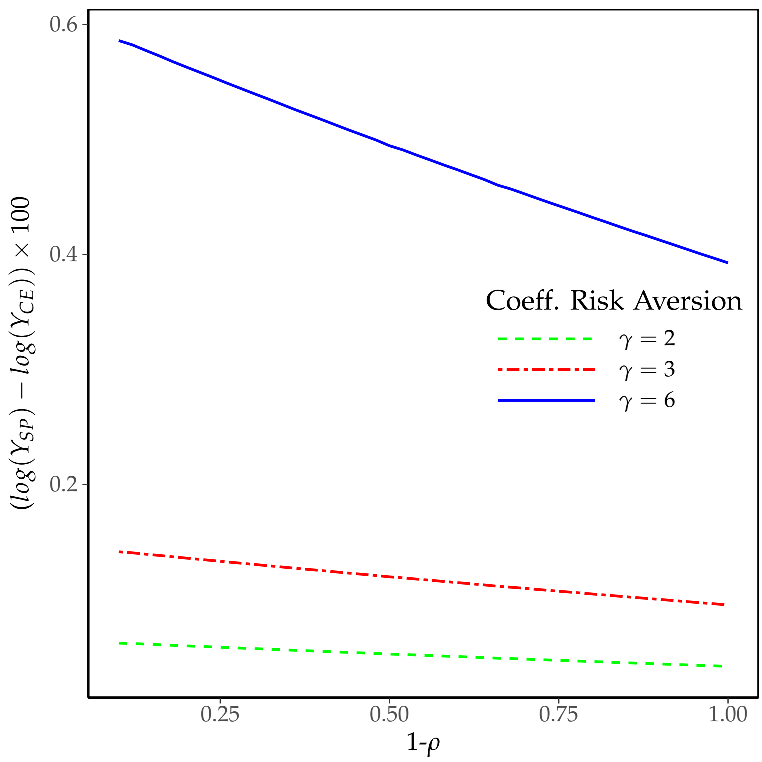


Figure 3: The Degree of Misallocation Across Industries

Notes: The figure plots the degree of misallocation. The degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}). The horizontal axis represents different values for $(1 - \rho)$. The three different lines represent different levels of risk aversion γ.

4.2 Risk, Import Penetration and the Misallocation of Workers

In our previous analysis we are silent about the sources of differences in the variance of permanent risk across industries. However, there is a growing number of studies that relate the degree of import penetration and trade exposure of an industry with the volatility of workers' earnings. An important paper in this literature is Krishna and Senses (2014) who document that a 10% increase in import penetration in an industry is associated with a 23% increase in the variance of permanent shocks to labor earnings.

As a consequence, as documented by a large body of literature on labor and trade, the increase in import competition has dramatically changed US labor markets. An important aspect is the increased importance of China as a competitive producer of manufactures after it entered the World Trade Organization. These authors document that the increase in import penetration of manufactures in the US accounts for a total loss of 12% of manufacturing employment in the United States.[10]

We use our framework to connect these two strands of the literature. We examine the output costs derived from the increase in import penetration in the tradable sector. In light of our model, everything else equal, the new cohort of risk averse workers tries to avoid the tradeable sector since the increase in risk due to trade openness makes the sector less attractive. We use our previous calibration but we now divide industries in two groups: "tradables" and "non-tradables". The tradable group comprises Durable Goods Manufacturing, Non-Durable Goods Manufacturing and Agricultural and Forestry. All other industries are included in non-tradables. The variances of the permanent shocks to earnings are 0.0061 and 0.0050 for the tradable sector and non-tradable sector, respectively. We interpret the allocations of our model with this parameterization as an initial steady state and entertain a trade reform to measure the change in the degree of misallocation. For this purpose, we use the estimates of Acemoglu, Autor, Dorn, Hanson, and Price (2016) who document an increase in the import penetration in the manufacturing sector of 7%. In addition, according to estimates of Krishna and Senses (2014), an increase of import penetration of 7%, corresponds to an increase in the variance of the permanent shock to labor earnings of the tradable sector of 16.1%. Thus, according to our estimates, the variance of the tradable sector would be 0.0070. Ceteris paribus, in the new equilibrium with a riskier tradable sector the model predicts an increase in the degree of misallocation and a decrease in the number of workers in the tradable sector.

Figure 4 shows the change in misallocation for different values of ρ and γ. We measure misallocation the same way as before: the percentage change of the com-

[10] Acemoglu, Autor, Dorn, Hanson, and Price (2016) report employment losses of about 2.2 million. Manufacturing employment in January of 1999 was about 17 million workers.

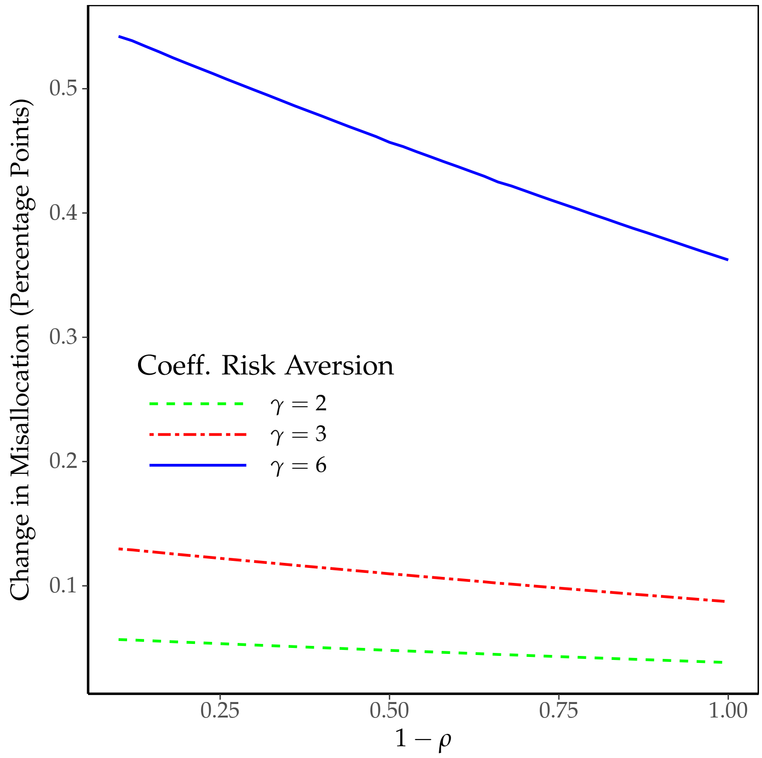


Figure 4: Import Penetration and Misallocation

Notes: The figure plots the degree of misallocation. The degree of misallocation is measured as the percentage deviation of output in a competitive equilibrium from output at the social optimum; i.e. by the percentage deviation of the competitive equilibrium output (Y_{CE}) from the first-best (Y_{SP}). The horizontal axis represents different values for $(1 - \rho)$. The three different lines represent different levels of risk aversion γ.

petitive equilibrium output from the first best. The figure plots the change in misallocation as trade opens. For example, if misallocation is 1% pre-trade and 1.5% post-trade, the change in misallocation is half a percentage point. For a given value of ρ and γ, there is an increase in misallocation following the trade reform. After the increase in trade openness the tradable industry is even riskier than the non-tradable industry. As a result, less workers enter the tradable sector, resulting in an allocation that is farther away from the first best than was pre-trade allocation. The magnitude of this increase in misallocation depends upon the values of ρ and γ. As the picture shows, the increase in misallocation can plausibly be as large as 0.7 percentage points. Changes of this magnitude require abilities to be highly dependent and workers to be quite risk-averse.

5 Conclusions

How does the lack of insurance markets to insure against worker's permanent earnings shocks affect their occupational choice and the allocation of human capital in an economy? What are the consequences for aggregate productivity? We have answered these questions by developing a Roy model of occupational choice. Risk averse workers choose an occupation based on the occupation-specific risk they face and on their comparative and absolute advantages. The tractability of the Frechet distribution allows for a closed-form solution of the competitive equilibrium allocation. In a competitive equilibrium, human capital is misallocated because workers avoid risky industries. The social planner allocates more workers to risky industries. The higher the risk aversion and the lower the degree of comparative advantage, the larger the misallocation. We perform two quantitative exercises to measure the size of misallocation. We estimate a permanent output loss of 0.6% due exclusively to market incompleteness.

We think this paper offers a new perspective for understanding the link between risk in labor markets and the aggregate levels of human capital. We focus on the interplay between abilities and risk. We abstract from many aspects of the labor market and the career choice of the individuals. For instance, we take earnings volatility as exogenous and we do not consider heterogeneity in risk aversion. For the sake of tractability and to obtain analytical expressions we also abstract from the career dynamics and the role that savings play in shaping the occupational choice. We also abstract from many barriers that surely affect the occupational choice and mobility of workers and that may interact with the lack of insurance. From this perspective, we think our measured misallocation can be a lower bound in our quantitative exercises. We hope our findings encourage future research that relaxes these assumptions.

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Appendix

A Frechet Marginal Distributions

Given a joint cumulative distribution $F_{x_R,x_S}(x_R, x_S)$ with support $(0,\infty) \times (0,\infty)$, the marginal distribution of x_S is given by

$$f_{x_S}(x_S) = \int_0^\infty f_{x_R,x_S}(x_R, x_S) dx_R \tag{11}$$

where the joint density is obtained from

$$f_{x_R,x_S}(x_R, x_S) = \frac{d^2}{dx_R dx_S} F_{x_R,x_S}(x_R, x_S)$$

For the Gumbel copula with Frechet distribution

$$F_{x_R,x_S}(x_R, x_S) = \exp\left(-\left(x_R^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{(1-\rho)}\right)$$

differentiating once with respect to x_S gives an expression for the joint density:

$$f_{x_S,x_S}(x_R, x_S) = \frac{d}{dx_R} \exp\left(-\left(x_R^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{(1-\rho)}\right)\left(x_R^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{-\rho} \alpha x_S^{-\frac{\alpha}{(1-\rho)}-1}$$

Using this in (1) gives

$$
\begin{aligned}
f_{x_S}(x_S) &= \int_0^\infty \frac{d}{dx_R} \exp\left(-\left(x_R^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{(}1-\rho)\right)\left(x_R^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{-\rho} \alpha x_S^{-\frac{\alpha}{(1-\rho)}-1} dx_R \\
&= \exp\left(-\left(\infty^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{(}1-\rho)\right)\left(\infty^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{-\rho} \alpha x_S^{-\frac{\alpha}{(1-\rho)}-1} \\
&- \exp\left(-\left(0^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{(}1-\rho)\right)\left(0^{-\frac{\alpha}{(1-\rho)}} + x_S^{-\frac{\alpha}{(1-\rho)}}\right)^{-\rho} \alpha x_S^{-\frac{\alpha}{(1-\rho)}-1} \\
&= \exp\left(-x_S^{-\alpha}\right) x_S^{-\frac{\alpha}{(1-\rho)}(-\rho)} \alpha x_S^{-\frac{\alpha}{(1-\rho)}-1} - 0 \\
&= \exp\left(-x_S^{-\alpha}\right) \alpha x_S^{-\alpha-1}
\end{aligned}
$$

This is the density of a Frechet distribution with cdf $F_{x_S} = \exp\left(-x_S^{-\alpha}\right)$. Therefore, the marginal distribution is independent of ρ. Note that the previous derivation assumed that $\rho \in (0,1)$.

B Proof of Proposition 2.1

Proof To verify that expression, note that $p_R = Prob(V_R > V_S)$. We can rewrite $V_i(x_i, w_i)$ as,

$$
V_i(x_i, w_i) = x_i^{1-\gamma} \int_{y \in \mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma}\, dF_i(y) \tag{12}
$$

Relabeling the integral as Ω_i, further rewrite $V_i(x_i, w_i)$ as $x_i^{1-\gamma}\Omega_i$. Note that $V_i(x_i, w_i) < 0$ for any $x_i, w_i > 0$. Since the occupational choice entails picking the maximum between $V_R(x_R, w_R)$ and $V_S(x_S, w_S)$, the choice is equivalent to choosing the minimum between $|V_R(x_R, w_R)|$ and $|V_S(x_S, w_S)|$. Therefore, $Pr(V_R > V_S) = Pr(|V_R| < |V_S|) = Pr(x_R^{1-\gamma}|\Omega_R| < x_S^{1-\gamma}|\Omega_S|) = Pr(x_R^{1-\gamma} < x_S^{1-\gamma}\frac{|\Omega_S|}{|\Omega_R|})$. Since $\gamma > 1$, [11]

[11] To understand the next equality, note that

$$
F_{x_R}(x_R, x_S) = \frac{d}{dx_R} \int_0^{x_R} \int_0^{x_S} f(z, w) dz dw = \int_0^{x_S} f(z, x_R) dz.
$$

$$Pr(V_R > V_S) = Pr\Big(x_R(|\Omega_R|/|\Omega_S|)^{1/(1-\gamma)} > x_S \Big) = \int_0^\infty F_{x_R}(x, x(|\Omega_R|/|\Omega_S|)^{1/(1-\gamma)})dx.$$

The derivative of the joint cumulative density function (2) with respect to x_R is,

$$F_{x_R}(x_R, x_S) = \exp\left\{ -\left[\sum_{i \in R,S} (T_i^{\alpha/(1-\rho)} x_i^{-\alpha/(1-\rho)}) \right]^{(1-\rho)} \right\} \left[\sum_{i \in R,S} (T_i^{\alpha/(1-\rho)} x_i^{-\alpha/(1-\rho)}) \right]^{-\rho} \alpha T_R^{\alpha/(1-\rho)} x_R^{-\alpha/(1-\rho)-1} \tag{13}$$

Substituting for $x_R = x$ and $x_S = x\frac{|\Omega_R|}{|\Omega_S|}^{1/(1-\gamma)}$, defining κ_i to be $\frac{|\Omega_R|}{|\Omega_i|}^{1/(1-\gamma)}$ and integrating gives,[12]

$$\int F_{x_R}(x, x(|\Omega_R|/|\Omega_S|)^{1/(1-\gamma)}dx =$$

$$= \int \exp\left\{ -\left[\sum_{i \in R,S} \left(\frac{x\kappa_i}{T_i} \right)^{-\alpha/(1-\rho)} \right]^{(1-\rho)} \right\} \left[\sum_{i \in R,S} \left(\frac{x\kappa_i}{T_i} \right)^{-\alpha/(1-\rho)} \right]^{-\rho} \alpha T_R^{\frac{\alpha}{(1-\rho)}} x^{-\frac{\alpha}{(1-\rho)}-1} dx =$$

$$= \int \exp\left\{ -\left[\sum_{i \in R,S} \left(\frac{x\kappa_i}{T_i} \right)^{-\alpha/(1-\rho)} \right]^{(1-\rho)} \right\} \left[\sum_{i \in R,S} \left(\frac{\kappa_i}{T_i} \right)^{-\frac{\alpha}{(1-\rho)}} \right]^{-\rho} \alpha T_R^{\frac{\alpha}{(1-\rho)}} x^{\frac{-\alpha}{(1-\rho)}(-\rho)} x^{-\frac{\alpha}{(1-\rho)}-1} dx =$$

$$= \left[\sum_{i \in R,S} \left(\frac{\kappa_i}{T_i} \right)^{-\frac{\alpha}{(1-\rho)}} \right]^{-1} T_R^{\frac{\alpha}{(1-\rho)}} \int \exp\left\{ -\left[\sum_{i \in R,S} T_i^{\frac{\alpha}{(1-\rho)}} \kappa_i^{-\frac{\alpha}{(1-\rho)}} x^{-\frac{\alpha}{(1-\rho)}} \right]^{(1-\rho)} \right\} \left[\sum_{i \in R,S} \left(\frac{\kappa_i}{T_i} \right)^{-\frac{\alpha}{(1-\rho)}} \right]^{(1-\rho)} \alpha x^{-\alpha-1} dx =$$

$$= \left[\sum_{i \in R,S} \left(\frac{\kappa_i}{T_i} \right)^{-\frac{\alpha}{(1-\rho)}} \right]^{-1} T_R^{\frac{\alpha}{(1-\rho)}} \int f(x)dx = T_R^{\frac{\alpha}{(1-\rho)}} \left[\sum_{i \in R,S} \left(\frac{\kappa_i}{T_i} \right)^{-\frac{\alpha}{(1-\rho)}} \right]^{-1} \tag{14}$$

Since κ_i equals $\frac{|\Omega_R|}{|\Omega_i|}^{1/(1-\gamma)}$ for $i = 1, 2$, substitution yields,

We use standard notation $f(x_R, x_S)$ for the joint probability density function.

[12]The lower and upper integration limits are understood to be 0 and ∞.

$$p_R = \frac{T_R^{\frac{\alpha}{(1-\rho)}} |\Omega_R(w_R)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{\sum_{i=1}^2 T_i^{\frac{\alpha}{(1-\rho)}} |\Omega_i(w_i)|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \tag{15}$$

C Proof of Proposition 2.2

Proof We denote by $\tilde{x}_i$ the average ability of a workers who choose occupation i. Given that shocks that workers experience after they have chosen an occupation are of mean equal to one, the amount of efficiency units in occupation i is given by $N_i = p_i \tilde{x}_i$. The distributional assumption on the joint distribution of $\boldsymbol{X} = (x_R, x_S)$ implies that the post-sorting distribution of abilities is also Fréchet.

To derive this result we begin by defining the extreme value $V^* = \min_i \left\{ x_i^{1-\gamma} |\Omega_i| \right\}$. As a result for a given $b > 0$, $Pr(V^* > b) = Pr(x_i^{1-\gamma}|\Omega_i| > b) = Pr(x_i^{1-\gamma} > b/|\Omega_i|)$ for all i, which in turn equals,

$$Pr\left(x_i < \left(\frac{b}{|\Omega_i|} \right)^{1/(1-\gamma)} \right) \text{ for all } i.$$

Using the joint cdf, that probability is given by,

$$\begin{aligned} F\left(\frac{b}{|\Omega_R|}, \frac{b}{|\Omega_S|} \right) &= \exp\left\{ - \left[\sum_{i \in R,S} T_i^{\frac{\alpha}{(1-\rho)}} \left(\frac{b}{|\Omega_i|} \right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}} \right]^{(1-\rho)} \right\} = \\ &= \exp\left\{ - \left[\sum_{i \in R,S} \left(T_i^{\frac{\alpha}{(1-\rho)}} |\Omega_i|^{\frac{\alpha}{(1-\rho)(1-\gamma)}} b^{\frac{-\alpha}{(1-\rho)(1-\gamma)}} \right) \right]^{(1-\rho)} \right\} = \\ &= \exp\left\{ - \left[\hat{T}^{(1-\rho)} \left(b^{\frac{-\alpha}{(1-\rho)(1-\gamma)}} \right)^{(1-\rho)} \right] \right\}. \end{aligned} \tag{16}$$

where $\hat{T} = \sum_{i \in R,S} T_i^{\frac{\alpha}{(1-\rho)}} |\Omega_i|^{\frac{\alpha}{(1-\rho)(1-\gamma)}}$. Since $Pr(V^* > b) = 1 - Pr(V^* < b)$, the cdf of V^* is given by,

$$Pr(V^* < b) = 1 - \exp\left\{ - \left[\hat{T}^{(1-\rho)} b^{-\alpha/(1-\gamma)} \right] \right\}. \tag{17}$$

Note that this is the distribution for the extreme value $V^* = x^{*1-\gamma}|\Omega^*| = \min_i x_i^{1-\gamma}|\Omega_i|$.

We are interested in the cdf of x^*, the distribution of abilities post-sorting. To obtain that distribution, note that $Pr(V^* > b) = Pr\big(x^* < \big(\frac{b}{|\Omega^*|}\big)^{1/(1-\gamma)}\big) = Pr(x^* < b^*)$ Using the first term in (16), that probability is given by,

$$Pr(x^* < b^*) = \exp\left\{-\Big[\sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\left(\frac{b}{|\Omega_i|}\right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\Big]^{(1-\rho)}\right\} =$$

$$= \exp\left\{-\Big[\sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\left(\frac{b}{|\Omega^*|}\right)^{-\frac{\alpha}{(1-\rho)(1-\gamma)}}\left(\frac{|\Omega^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\Big]^{(1-\rho)}\right\} =$$

$$= \exp\left\{-\Big[\sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\left(\frac{b}{|\Omega^*|}\right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}}\left(\frac{|\Omega^*|}{|\Omega_i|}\right)^{-\alpha/\rho(1-\gamma)}\Big]^{(1-\rho)}\right\} =$$

$$= \exp\left\{-\Big[\sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\left(\frac{|\Omega^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{(1-\rho)(1-\gamma)}} {b^*}^{\frac{-\alpha}{(1-\rho)}}\Big]^{(1-\rho)}\right\}$$

$$= \exp\left\{-\Big[T^* {b^*}^{\frac{-\alpha}{(1-\rho)}}\Big]^{(1-\rho)}\right\}$$

$$= \exp\left\{-\Big[{T^*}^{\frac{-(1-\rho)}{\alpha}} b^*\Big]^{-\alpha}\right\} \tag{18}$$

where ${T_i}^* = \sum_{i\in R,S} T_i^{\frac{\alpha}{(1-\rho)}}\left(\frac{|{\Omega_i}^*|}{|\Omega_i|}\right)^{\frac{-\alpha}{\rho(1-\gamma)}}$.

Equation (18) shows that the distribution of x^*, the ability of workers who have chosen an occupation, is Fréchet. Its shape parameter is equal to α and its scale parameter is ${T^*}^{\frac{(1-\rho)}{\alpha}}$. The mean of this distribution is ${T^*}^{\frac{(1-\rho)}{\alpha}}\Gamma(1-\frac{1}{\alpha})$.

By letting $|{\Omega_i}^*| = |\Omega_i|$, we have that

$$T_i^* = T_i^{\frac{\alpha}{(1-\rho)}}/p_i$$

. Thus, the mean of that distribution can be written as $T_i p_i^{\frac{-(1-\rho)}{\alpha}}\Gamma(1-\frac{1}{\alpha})$. For occupation R, it is given by,

$$\tilde{x}_R = E(x_R) = T_R p_R^{\frac{-(1-\rho)}{\alpha}}\Gamma(1-1/\alpha), \tag{19}$$

And for occupation S by,

$$\tilde{x}_S = E(x_S) = T_S p_S^{\frac{-(1-\rho)}{\alpha}} \Gamma(1 - 1/\alpha), \tag{20}$$

Once we have $E(\tilde{x}_1)$ and $E(\tilde{x}_2)$ the result follows:

$$N_i = p_i \tilde{x}_i = T_i p_i^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma(1 - 1/\alpha), \tag{21}$$

D Proof of Proposition 2.3

To begin note that from by combining 2.1 and 2.2, N_i equals

Proof

$$N_i = T_i p_i^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) = T_i \left[\frac{T_i^{\frac{\alpha}{(1-\rho)}} \Omega_i^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{T_R^{\frac{\alpha}{(1-\rho)}} \Omega_R^{\frac{\alpha}{(1-\rho)(1-\gamma)}} + T_S^{\frac{\alpha}{(1-\rho)}} \Omega_S^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \right]^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) =$$

$$T_i \left[\sum_{j=1}^{2} \left(\frac{T_j}{T_i}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_j}{\Omega_i}\right)^{\frac{\alpha}{(1-\rho)(1-\gamma)}} \right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{22}$$

Also note that the ratio of the two labor inputs in efficiency units is,

$$\frac{N_R}{N_S} = \frac{T_R}{T_S} \left(\frac{T_R^{\frac{\alpha}{(1-\rho)}} \Omega_R^{\frac{\alpha}{(1-\rho)(1-\gamma)}}}{T_S^{\frac{\alpha}{(1-\rho)}} \Omega_S^{\frac{\alpha}{(1-\rho)(1-\gamma)}}} \right)^{\frac{\alpha-(1-\rho)}{\alpha}} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}}$$

$$= \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{w_R^{1-\gamma} E_R}{w_S^{1-\gamma} E_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}} \tag{23}$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)})$. In equilibrium, wages are equal to the marginal products of the two types of labor. Given our aggregate technology,

$$Y = [\theta N_1^{\nu} + (1-\theta) N_2^{\nu}]^{1/\nu} \tag{24}$$

we have that

$$w_R = 1/\nu\, [\theta N_1^\nu + (1-\theta) N_2^\nu]^{1/\nu - 1}\, \theta N_1^{\nu-1}$$

and

$$w_R = 1/\nu\, [\theta N_1^\nu + (1-\theta) N_2^\nu]^{1/\nu - 1}\, (1-\theta) N_2^{\nu-1}.$$

Thus,

$$\frac{w_R}{w_S} = \left(\frac{\theta}{1-\theta}\right) \left(\frac{N_R}{N_S}\right)^{\nu-1} \tag{25}$$

Substituting (25) into (23), we get

$$\frac{N_R}{N_S} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)}} \left(\frac{N_R}{N_S}\right)^{-(\nu-1)\frac{(1-\rho)-\alpha}{(1-\rho)}} \left(\frac{E_R}{E_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}} \tag{26}$$

Simplifying

$$\frac{N_R}{N_S} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-(1-\rho)}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_R}{E_S}\right)^{\frac{\alpha-(1-\rho)}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \tag{27}$$

Note from (22) that N_R is,

$$N_R = T_R \left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_S}{\Omega_R}\right)^{\frac{\alpha}{(1-\rho)(1-\gamma)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{28}$$

$$= T_R \left[1 + \left(\frac{T_S}{T_R}\left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{1}{(\gamma-1)}}\right)^{\frac{\alpha}{(1-\rho)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{29}$$

and from (23)

$$\frac{N_R}{N_S} = \frac{T_R}{T_S}\left(\frac{T_S}{T_R}\left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{1}{(\gamma-1)}}\right)^{\frac{(1-\rho)-\alpha}{(1-\rho)}} \tag{30}$$

so that,

$$\frac{T_S}{T_R}\left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{1}{(\gamma-1)}} = \left(\frac{T_S}{T_R}\frac{N_R}{N_S}\right)^{\frac{(1-\rho)}{(1-\rho)-\alpha}}. \tag{31}$$

Substituting back into (29),

$$N_R = T_R\left[1+\left(\frac{T_S N_R}{T_R N_S}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) =$$

$$\left[1+\left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\left(\frac{N_R}{N_S}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \tag{32}$$

Substituting for the value of the ratio of labor inputs given by (27)

$$N_R = T_R\left[1+\left(\frac{T_S}{T_R}\left(\frac{T_S}{T_R}\right)^{\frac{-\alpha}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{\theta}{1-\theta}\right)^{\frac{\alpha-(1-\rho)}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{E_R}{E_S}\right)^{\frac{\alpha-(1-\rho)}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}}\right)^{\frac{\alpha}{(1-\rho)-\alpha}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \tag{33}$$

Further simplification gives,

$$N_R = T_R\left[1+\left(\frac{T_S}{T_R}\right)^{\frac{\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha)((1-\rho)-\alpha)}}\left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu((1-\rho)-\alpha)+\alpha}}\left(\frac{E_S}{E_R}\right)^{\frac{\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}}\right]^{\frac{(1-\rho)-\alpha}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right) \tag{34}$$

Similarly for N_S we have,

$$N_S = T_S \left[1 + \left(\frac{T_S}{T_R}\right)^{\frac{-\alpha\nu((1-\rho)-\alpha)}{(\nu((1-\rho)-\alpha)+\alpha)((1-\rho)-\alpha)}} \left(\frac{1-\theta}{\theta}\right)^{\frac{-\alpha}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_S}{E_R}\right)^{\frac{-\alpha}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \right]^{\frac{(1-\rho)-\alpha}{\alpha}} \Gamma\left(1-\frac{1}{\alpha}\right) \tag{35}$$

By substituting the expressions for N_R and N_S into (24) we obtain the competitive equilibrium level of output Y_{CE}.

E The Compensation for Risk in the Labor Market

Differences in risk across occupations imply that workers face a risk-return trade-off in the labor market. This section derives the equilibrium wage differential across occupations and shows how it depends on agents' risk aversion, workers' comparative advantage and the risk spread across occupations.

E.0.1 The Wage Premium and the Compensation for Risk

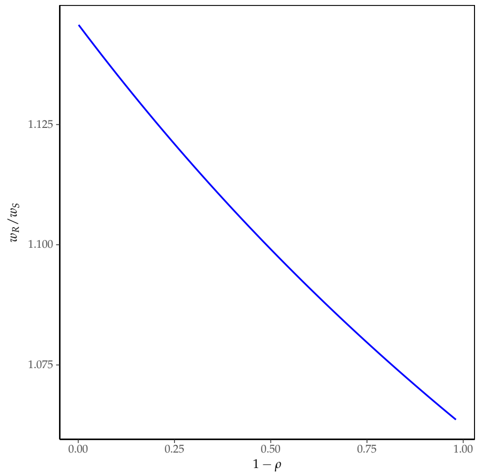
In equilibrium the ratio of wage rates or prices is the ratio of marginal productivities. Using 1 and can be written as,

$$WP = \frac{w_R}{w_S} = \frac{\theta}{1-\theta}\left(\frac{N_R}{N_S}\right)^{\nu-1}. \tag{36}$$

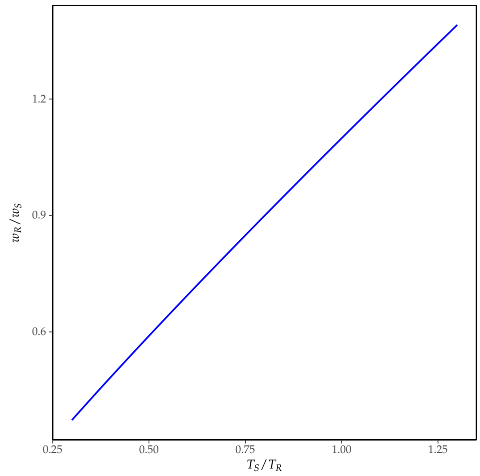
Using (27) to substitute for N_R/N_S we have that

$$WP = \frac{w_R}{w_S} = \left(\frac{1-\theta}{\theta}\right)^{-\frac{(1-\rho)}{\nu((1-\rho)-\alpha)+\alpha}} \left(\frac{E_S}{E_R}\right)^{\frac{(\alpha-(1-\rho))(1-\nu)}{(\nu((1-\rho)-\alpha)+\alpha)(1-\gamma)}} \left(\frac{T_R}{T_S}\right)^{\frac{\alpha(\nu-1)}{\nu((1-\rho)-\alpha)+\alpha}} \tag{37}$$

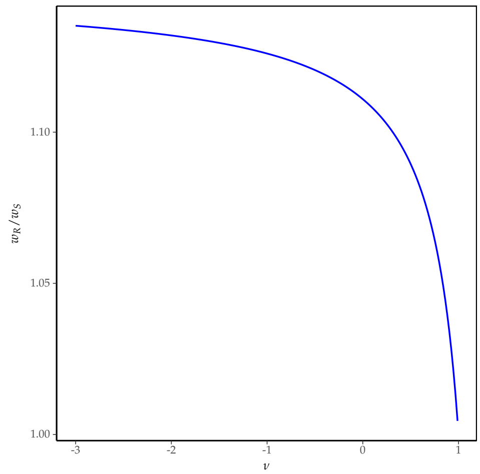
The ratio of wages has three components. The first term is related to the shape of the aggregate technology. Everything else constant, wages rise in occupation R if θ



(a) Wage Premium and $(1-\rho)$.



(b) Wage Premium and T_S/T_R.



(c) Wage Premium and ν.

Figure 5: The three figures show how wage premium of the risky relative to the safe industry, varies for different values of three parameters: (a) ρ, (b) T_S/T_R, and (c) ν.

falls. The second term, represents the compensation for risk. This premium rises with γ and equals zero when $\gamma = 0$. It also rises with the spread between the variances of the idiosyncratic shocks. The third term represents the influence of the ratio of the means of the distribution of abilities on the ratio of wages. If ability for occupation R is more abundant (T_R is higher) its price drops, everything else constant.

How do the different parameters affect the relative price of the two types of human capital? The answer is shown in Figure 5. We begin by analyzing the changes in the ratio of wage rates w_R/w_S for different values of $(1-\rho)$. This parameter governs the degree of dependence between the abilities of workers, also interpreted as the degree of comparative advantage. When ρ approaches one (zero) it means that the ability draws of a worker are very dependent (non-dependent). In other words, when ρ is close to one, if a worker is good at performing one occupation there is also a high probability of being also good at the other occupation. We can think of ρ approaching one as the limiting case in which there is only one ability to perform both occupations or, just absolute advantage. As it is clear in the picture, the lower ρ is the lower the relative wage rate in occupation R. The reason in this case is very simple, when ρ is low then there is more selection in equilibrium. It is always the case that less workers will choose the risky occupation (because they are risk averse), but the lower the ρ the more selected they will be and thus with higher mean ability conditional on choosing the risky occupation (and thus efficiency units). Since the technology exhibits decreasing returns at the occupational level then the lower the relative wage.

The second picture plots the ratio of wages as the ratio T_S/T_R changes. As T_S/T_R increases, the abilities of occupation R are relatively scarce and thus, everything else equal, one unit of human capital of occupation R is relatively more expensive.

The third picture shows the ratio of wage rates for different values of ν, starting with negative values – more complementarity across the two occupations in production – up to one (perfect substitutes). The more substitutable the occupations are when producing output, the lower the price of one unit of human capital in occupation R relative to occupation S. When occupations are complementary, it is necessary to have workers in both occupations. The only way to attract workers to the risky occupations is a high wage. As the degree of substitution rises, the economy can employ workers in the second occupation without lowering output as much. The need for a high premium is therefore reduced.

E.0.2 The Earnings Premium

As opposed to the ratio of wages, the earnings premium is observed in the data. It's defined as the ratio of average earnings across the two occupations:

$$EP = \frac{\frac{w_R N_R}{p_R}}{\frac{w_S N_S}{p_S}} \tag{38}$$

From 2.2 we have that

$$p_i = \left(\frac{N_i}{T_i \Gamma(1 - \frac{1}{\alpha})} \right)^{\frac{\alpha}{\alpha-(1-\rho)}}. \tag{39}$$

Thus,

$$\frac{p_S}{p_R} = \left(\frac{T_R}{T_S} \right)^{\frac{\alpha}{\alpha-(1-\rho)}} \left(\frac{N_S}{N_R} \right)^{\frac{\alpha}{\alpha-(1-\rho)}}. \tag{40}$$

Substituting,

$$EP = \frac{w_R}{w_S} \left(\frac{T_R}{T_S} \right)^{\frac{\alpha}{\alpha-(1-\rho)}} \left(\frac{N_R}{N_S} \right)^{\frac{-(1-\rho)}{\alpha-(1-\rho)}}. \tag{41}$$

By using (25) we now have that

$$EP = \frac{\theta}{1-\theta} \left(\frac{N_R}{N_S} \right)^{\frac{((1-\rho)-\alpha)(\nu-1)+(1-\rho)}{(1-\rho)-\alpha}}. \tag{42}$$

From (27)

$$\frac{N_R}{N_S} = \left(\frac{T_S}{T_R} \right)^{\frac{-\alpha}{(1-\rho)}} \left(\frac{\Omega_R}{\Omega_S} \right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}}. \tag{43}$$

Substituting,

$$EP = \frac{w_R}{w_S} \left(\frac{\Omega_R}{\Omega_S} \right)^{\frac{1}{\gamma-1}}. \tag{44}$$

$$EP = \frac{w_R}{w_S} \left(\frac{w_R^{1-\gamma} E_R}{w_S^{1-\gamma} E_S} \right)^{\frac{1}{\gamma-1}}. \tag{45}$$

$$EP = \left(\frac{E_R}{E_S} \right)^{\frac{1}{\gamma-1}}. \tag{46}$$

Interestingly, the earnings premium only depends on the parameters that govern the risk premium, i.e. the relative variance of earnings shocks and the coefficient of risk aversion. As expected, the higher the value of γ the higher the ratio of earnings. This is clearly depicted in 6. Everything else equal the higher the risk aversion, the higher the compensation she/he requires to choose the risky occupation R. For a fixed risk aversion parameter, the higher the volatility of shocks of occupation R relative to S, the higher the compensation for the risk workers face.

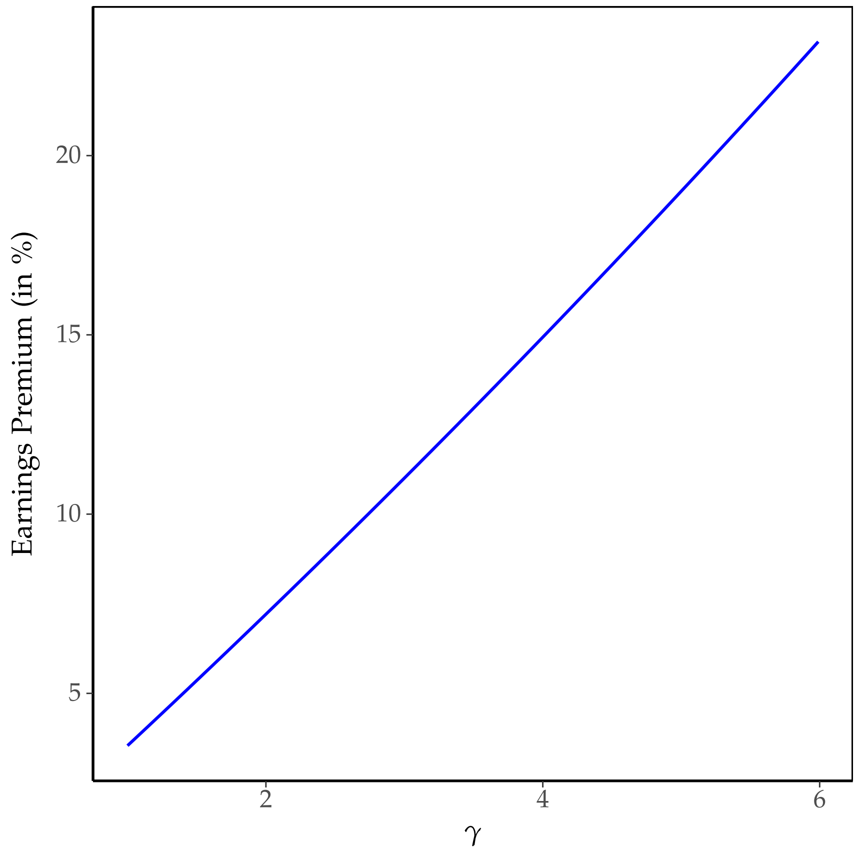


Figure 6: Risk Aversion and the Earnings Premium

Notes: The figure shows how the earnings premium defined as the average earnings of the risky occupation relative the safe occupation varies with the risk aversion coefficient.

F The Social Planner's Allocation

We equalize the first order conditions for this problem render (note that the term containing the Γ function cancels out because it is a constant):

$$\theta T_R^{\nu} \left(p_R^{SP} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}-1} = T_S^{\nu}(1-\theta) \left(p_S^{SP} \right)^{\nu \frac{\alpha-(1-\rho)}{\alpha}-1} \tag{47}$$

Since the two masses have to add up to one, we get that

$$p_R^{SP} = \frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \tag{48}$$

and

$$p_S^{SP} = \frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1}. \tag{49}$$

Plugging back into the definition of efficiency units we get the allocation of efficiency units chosen by the social planner:

$$N_R^{SP} = T_R \left[\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right]^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{50}$$

$$N_S^{SP} = T_S \left[\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right]^{\frac{\alpha-(1-\rho)}{\alpha}} \Gamma\left(1 - \frac{1}{\alpha}\right) \tag{51}$$

Given the labor inputs chosen by the planner, the efficient level of output is

$$\begin{aligned} Y_{SP} = \Bigg[\theta T_R^{\nu} & \left(\frac{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}}}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right)^{\nu\frac{\alpha-(1-\rho)}{\alpha}} + \\ (1-\theta) T_R^{\nu} & \left(\frac{1}{\frac{(1-\theta)}{\theta}^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \frac{T_S}{T_R}^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} + 1} \right)^{\nu\frac{\alpha-(1-\rho)}{\alpha}} \Bigg]^{1/\nu} \Gamma\left(1 - \frac{1}{\alpha}\right) \end{aligned} \tag{52}$$

G Corrective Taxation

Proposition G.1 *If wages in occupations R and S are taxed by occupation-specific taxes* τ_R *and* τ_S*, respectively, the social planner's allocation is achieved by setting taxes such that*

$$\frac{1-\tau_R}{1-\tau_S} = \left(\frac{E_R}{E_S}\right)^{\frac{1}{\gamma-1}}$$

where $E_i = \mathbb{E}(e^{y_i(1-\gamma)})$*. Furthermore, if taxes are chosen so that government's budget* *remains balanced, the tax rates are given by*

$$\tau_R = \frac{1 - \left(\frac{E_S}{E_R}\right)^{\frac{1}{1-\gamma}}}{1 + \left(\frac{T_S}{T_R}\right)^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \left(\frac{E_S}{E_R}\right)^{\frac{1}{1-\gamma}}}$$

and

$$\tau_S = -\left(\frac{T_S}{T_R}\right)^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \tau_R$$

The most interesting aspect to note is that the ratio of tax rates given in the proposition, that is $\frac{1-\tau_R}{1-\tau_S}$, is the same as the one given in the earnings premium in equation (46). The reason is simple. The taxes try to correct the misallocation generated in competitive equilibrium by encouraging more workers to choose the risky occupation. The way to do that is to tax relatively more the workers that choose the safe occupation. As it clearly transpires from the expression, the higher the variance of income shocks of occupation R relative to occupation S, everything else equal, the higher is the ratio is $\left(\frac{E_R}{E_S}\right)^{\frac{1}{\gamma-1}}$. Thus, the higher is the relative tax rate on earnings of the workers of the safe occupation (τ_S) relative to the workers in the risky occupation (τ_R). Similarly, for the same variance of the shocks, the higher the risk aversion parameter, the higher the degree of misallocation and thus the higher the tax rates on earnings of workers in the safe occupation relative to the workers in the risky occupation that is needed to obtain the social planner allocation.

Proof If a firm pays employee in occupation i a wage of w_i, the after-tax wage is $(1-\tau_i)w_i$. Using 2.1 and 2.2, $\frac{N_R}{N_S}$ equals

$$\frac{N_R}{N_S} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{\Omega_R}{\Omega_S}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)(1-\gamma)}} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{(1-\rho)}} \left(\frac{(1-\tau_R)w_R E_R^{\frac{1}{1-\gamma}}}{(1-\tau_S)w_S E_S^{\frac{1}{1-\gamma}}}\right)^{\frac{\alpha-(1-\rho)}{(1-\rho)}} \tag{53}$$

where the second equality follows from the definition of $\Omega_i \int_{y\in\mathbb{Y}} \frac{(e^y w_i)^{1-\gamma}}{1-\gamma}\, dF_i(y)$. Because of the aggregate CES technology, wages have to satisfy

$$\frac{w_R}{w_S} = \frac{\theta N_R^{\nu-1}}{(1-\theta)N_S^{\nu-1}} \tag{54}$$

Substituting (54) to the right side of (53) and simplifying gives

$$\frac{N_R}{N_S} = \left(\frac{T_R}{T_S}\right)^{\frac{\alpha}{\alpha-\nu(\alpha-(1-\rho))}} \left(\frac{1-\tau_R}{1-\tau_S}\frac{\theta}{1-\theta}\frac{E_R^{\frac{1}{1-\gamma}}}{E_S^{\frac{1}{1-\gamma}}}\right)^{\frac{\alpha-(1-\rho)}{\alpha-\nu(\alpha-(1-\rho))}} \tag{55}$$

From (50) and (51) we have

$$\frac{N_R^{SP}}{N_S^{SP}} = \left(\frac{T_S}{T_R}\right)^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha-(1-\rho)}{\nu(\alpha-(1-\rho))-\alpha}} \tag{56}$$

Setting the right hand side of (55) equal to the right hand side of (56) gives the expression in the first part of Proposition G.1.

If we additionally require that government budget is balanced, we have that

$$\tau_1 N_R w_R + \tau_2 N_S w_S = 0 \tag{57}$$

or, in a more convenient form as

$$\tau_2 = \frac{-\tau_1 N_R w_R}{N_S w_S} \tag{58}$$

Substituting in the wage ratio given by equation (54) and evaluating this at $\frac{N_R^{SP}}{N_S^{SP}}$ (given by equation (56)) leads to

$$\tau_2 = -\tau_1 \left(\frac{T_S}{T_R}\right)^{\frac{\alpha\nu}{\nu(\alpha-(1-\rho))-\alpha}} \left(\frac{1-\theta}{\theta}\right)^{\frac{\alpha}{\nu(\alpha-(1-\rho))-\alpha}} \tag{59}$$

The expression for τ_2 given by (59) can be used to obtain an expression for τ_1 from

$$\frac{1-\tau_1}{1-\tau_2} = \left(\frac{E_S}{E_R}\right)^{\frac{1}{1-\gamma}}$$

Doing so leads to the expression in the latter part of Proposition G.1 which only depends on the primitives of the model.